Bilad Alrafidain University College

Electric Power Techniques Engineering Department

Control Systems Analysis

Fourth Stage

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Control Systems Analysis

Course Contents

- Introduction to Control System.
- Transfer Function.
- Time Domain Analysis.
- Stability Analysis.
- Root Locus Method.
- Frequency Domain Analysis.
- Compensator Lead Network.
- Compensator Lag Network.
- PID Controllers.
- State Space Theory.
- State Space Representation.

Lecture Three

Transfer Function

Poles & Zeros of a Transfer Function

Poles : Poles of a Transfer Function are the frequencies (Values of s) for which the DENOMINATOR of the Transfer Function becomes ZERO.

Zeros : Zeros of a Transfer Function are the frequencies (Values of s) for which the NUMERATOR of the Transfer Function becomes ZERO.

The general form of the Transfer Function :

$$T.F = \frac{(s-z_1)(s-z_2)(s-z_3)\ \dots\ (s-z_n)}{(s-p_1)(s-p_2)(s-p_3)\ \dots\ (s-p_n)}$$

Zeros : $s = z_1, z_2, z_3, \dots\ z_n$

Poles : $s = p_1, p_2, p_3, \dots\ p_n$

Example 1 : Find the Transfer Function of the system given by:

$$\frac{d^2y(t)}{dt^2} + 7\frac{dy(t)}{dt} + 12y(t) = \frac{dx(t)}{dt} + 2x(t)$$

Where : $x(t) is\ the\ input\ \&\ y(t)\ is\ the\ output$

Transfer Function $G(S) = \frac{Y\ (S)}{X\ (S)} : \frac{NUMERATOR}{DENOMINATOR}$

Solution : Taking Laplace Transform to the components of the Transfer Function:

$$s^2.Y(s) + 7s.Y(s) + 12Y(s) = s.X(s) + 2.X(s)$$

$$Y(s).[s^2 + 7s + 12] = X(s)\ .\ [s+2]$$

$$\frac{Y(s)}{X(s)} = \frac{s+2}{s^2+7s+12}$$

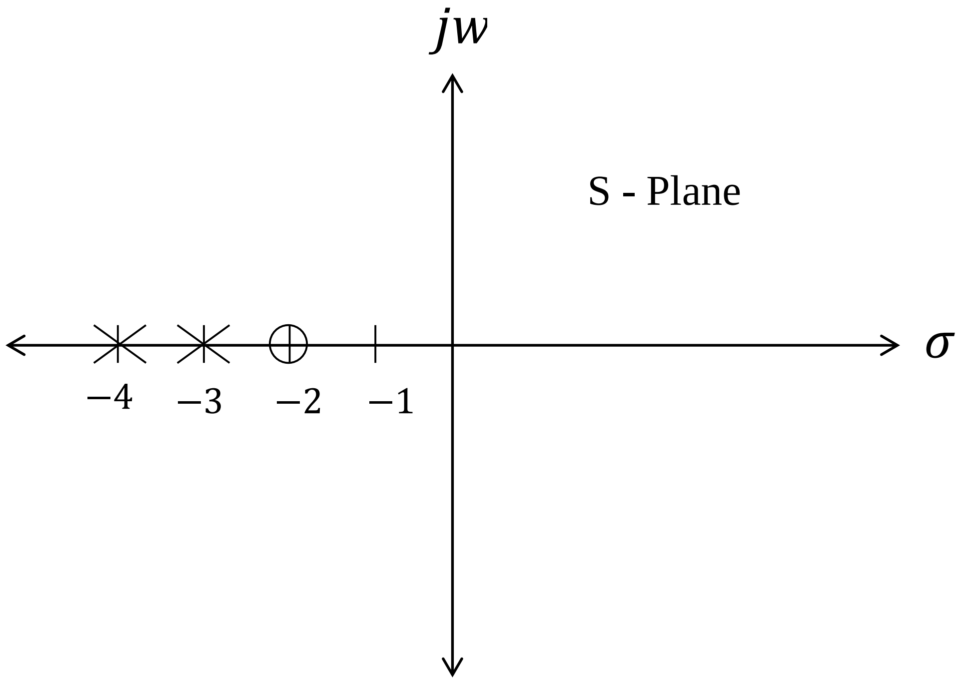
$$G(s) = \frac{s+2}{(s+3)(s+4)}$$

Now the Poles & Zeros of the Transfer Function $G(s) = \frac{s+2}{(s+3)(s+4)}$ are :

Zeros : $s = -2$

Poles : $s = -3, s = -4$

Let us now draw the (Pole – Zero Diagram) which is a plot on (S-plane) represents the locations of Poles and Zeros of a Transfer Function. In the (Pole – Zero Diagram) the Poles are represented by (X) and the Zeros represented by (O).



Example 2 : Find the Poles & Zeros for the following Transfer Function and then plot them on the (S-Plane)?

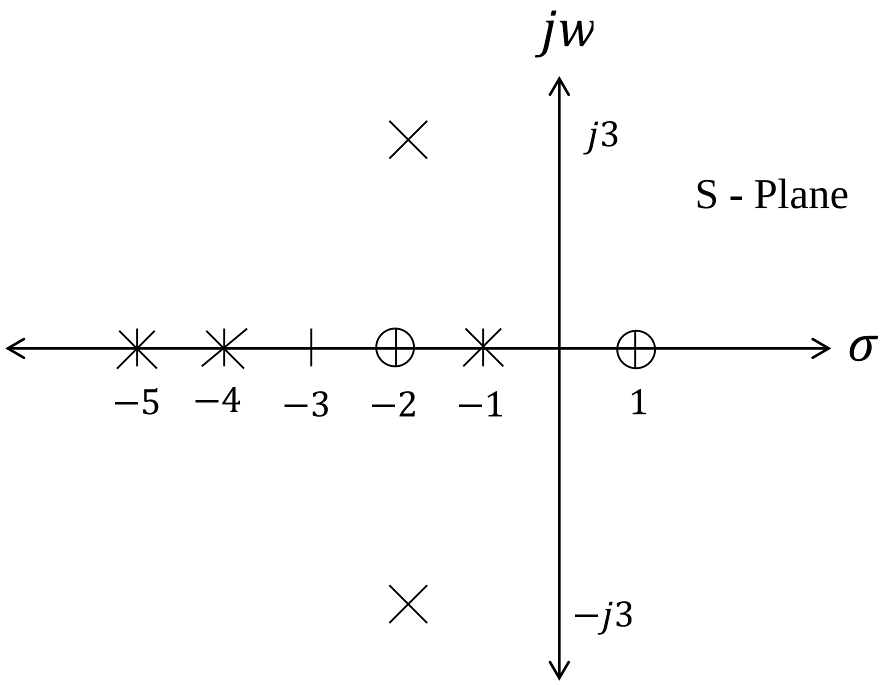
$$G(S) = \frac{(s+2)(s-1)}{(s+1\,)(s+4)(s+5)(s+2+j3)(s+2-j3)}$$

Solution :

Zeros : $s = \ -2\ , s = \ 1$

Poles : $s = \ -1, s = -4, s = \ -5, s = -2 - j3, s = \ -2 + j3$

Let us now draw the (Pole – Zero Diagram) which is a plot on (S-plane) represents the locations of Poles and Zeros of a Transfer Function. In the (Pole – Zero Diagram) the Poles are represented by (X) and the Zeros represented by (O).



Homework

Q1 / Find the Poles & Zeros for the following Transfer Function and then plot them on the (S-Plane)?

$$G(S) = \frac{(2s + 6)(s+2)}{s.(s+1)(s+4)}$$

$$G(S) = \frac{5}{s^3 + 6s^2 + 11s + 6}$$

$$G(S) = \frac{(s+1.5)(s+3+j2)(s+3-j2)}{(s+1)(s+4)(s+10)}$$