Bilad Alrafidain University College

Electric Power Techniques Engineering Department

Control Systems Analysis

Fourth Stage

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Lecture Fifteen

Frequency Domain Analysis (Bode Diagram)

Assistant Lecturer. Ibrahim Ismail

Introduction to (Bode Diagram)

- Bode diagrams are also known as logrithmic plots
- Bode diagram consists of two graphs:
  - Magnitude measured in decibel (dB)
  - Phase angle measured in degree
- The base of the logarithm is 10
- The curves are drawn on semilog paper, using the log scale for frequency

Main advantage of bode diagram:

- Multiplication of magnitudes can be converted into addition
- Approximation by straight-line asymptotes is sufficient if only rough information on the frequency-response characteristics

Properties of Log

$$A \times \log_d C = L \quad \Rightarrow \log_d C^A = L$$

$$\Rightarrow C^A = d^L, \qquad C = d^{(L/A)}$$

$$\log A = \log_{10} A \quad \Rightarrow \text{base of } 10$$

$$\log_{10} 1 = 0 \qquad \log_{10} 0 = -\infty$$

$$C \log AB = C \log A + C \log B$$

$$C \log \tfrac{A}{B} = C \log A - C \log B$$

$$C \log \tfrac{1}{B} = C \log B^{-1} = -C \log B$$

Bode Diagrams

$F(s)$	$F(j\omega)$	Slope $(dB/decade)$	Angle (deg)
$\frac{1}{s}$	$\frac{1}{j\omega}$	-20	-90
s	$j\omega$	$+20$	$+90$
$\frac{1}{s^n}$	$\frac{1}{(j\omega)^n}$	$-20 \times n$	$-90 \times n$
s^n	$(j\omega)^n$	$+20 \times n$	$+90 \times n$

Bode Plot in Five Steps

Bode diagram is plotted for $H(s)\,G(s)$

$$H(s)\,G(s) = \frac{L \times s^{k_2} \times (s+A)}{s^{k_1}(s+B)(s+C)}$$

Step 1. Rewrite $H(s)\,G(s)$ and substitute for s by $j\omega$

$$H(j\omega)\,G(j\omega) = \frac{L \times (j\omega)^{k_2} \times (j\omega + A)}{(j\omega)^{k_1}(j\omega + B)(j\omega + C)}$$

keep the part multiplied by s^0 equal 1

$$H(j\omega)\,G(j\omega) = \frac{L \times A}{B \times C}\,\frac{(j\omega)^{k_2-k_1} \times \left(j\frac{\omega}{A}+1\right)}{\left(j\frac{\omega}{B}+1\right)\left(j\frac{\omega}{C}+1\right)}$$

$$H(j\omega)\,G(j\omega) = K\,\frac{(j\omega)^{k_2-k_1} \times \left(j\frac{\omega}{A}+1\right)}{\left(j\frac{\omega}{B}+1\right)\left(j\frac{\omega}{C}+1\right)}, \qquad K = \frac{L \times A}{B \times C}$$

Bode Plot in Five Steps

Step 2. Draw table as below

Factor	Type	(Corner/Break) frequency	Slope
$(j\omega)^{k_2-k_1}$	-	-	Starting slope $+20 \times (k_2 - k_1)$
$(j\frac{\omega}{A}+1)$	zero	A	$+20$
$(j\frac{\omega}{B}+1)$	pole	B	-20
$(j\frac{\omega}{C}+1)$	pole	C	-20

Bode Plot in Five Steps

Step 3. Starting frequency

$$\omega_{st} = \frac{\text{Least value of corner frequency}}{10} = \frac{\min\{A, B, C\}}{10}$$

Step 4. Starting magnitude $= 20\log(K) + 20 \times (k_2 - k_1)\log(\omega_{st})$

Step 5. Phase angle

$$\varphi = \tan^{-1}\left(\frac{0}{K}\right) + \underbrace{\operatorname{sgn}(k_2 - k_1)}_{-\text{ or }+}\tan^{-1}\left(\frac{\omega}{0}\right) + \tan^{-1}\left(\frac{\omega}{A}\right) - \tan^{-1}\left(\frac{\omega}{B}\right) - \tan^{-1}\left(\frac{\omega}{C}\right)$$

Example. 1. (Bode Diagram)

Draw the Bode plot of

$$H(s)\,G(s) = \frac{300s\,(s+5)}{(s+1)\,(s+30)}$$

Step 1. Rewrite $H(s)\,G(s)$ and replace s by $j\omega$

$$H(j\omega)\,G(j\omega) = \frac{300j\omega\,(j\omega+5)}{(j\omega+1)\,(j\omega+30)}$$

Next,

$$H(j\omega)\,G(j\omega) = \frac{300 \times 5}{1 \times 30} \times \frac{j\omega\,(j\frac{\omega}{5}+1)}{(j\frac{\omega}{1}+1)\,(j\frac{\omega}{30}+1)}$$

Thus,

$$H(j\omega)\,G(j\omega) = 50 \times \frac{j\omega\,(j\frac{\omega}{5}+1)}{(j\frac{\omega}{1}+1)\,(j\frac{\omega}{30}+1)}$$

Example. 1. (Bode Diagram)

Step 2. Draw table as below

Factor	Type	(Corner/Break) frequency	Slope
$(j\omega)$	zero	-	+20 (Starting slope)
$(j\frac{\omega}{5}+1)$	zero	5	+20
$(j\frac{\omega}{1}+1)$	pole	1	−20
$(j\frac{\omega}{30}+1)$	pole	30	−20

Step 3. Starting frequency

$$\omega_{st} = \frac{\text{Least value of corner frequency}}{10} = \frac{1}{10} = 0.1$$

Example. 1. (Bode Diagram)

Step 4. Starting magnitude

$$= 20\log(K) + 20 \times (k_2 - k_1)\log(\omega_{st})$$
$$= 20\log(50) + 20 \times 1 \times \log(0.1) = 13.9794$$

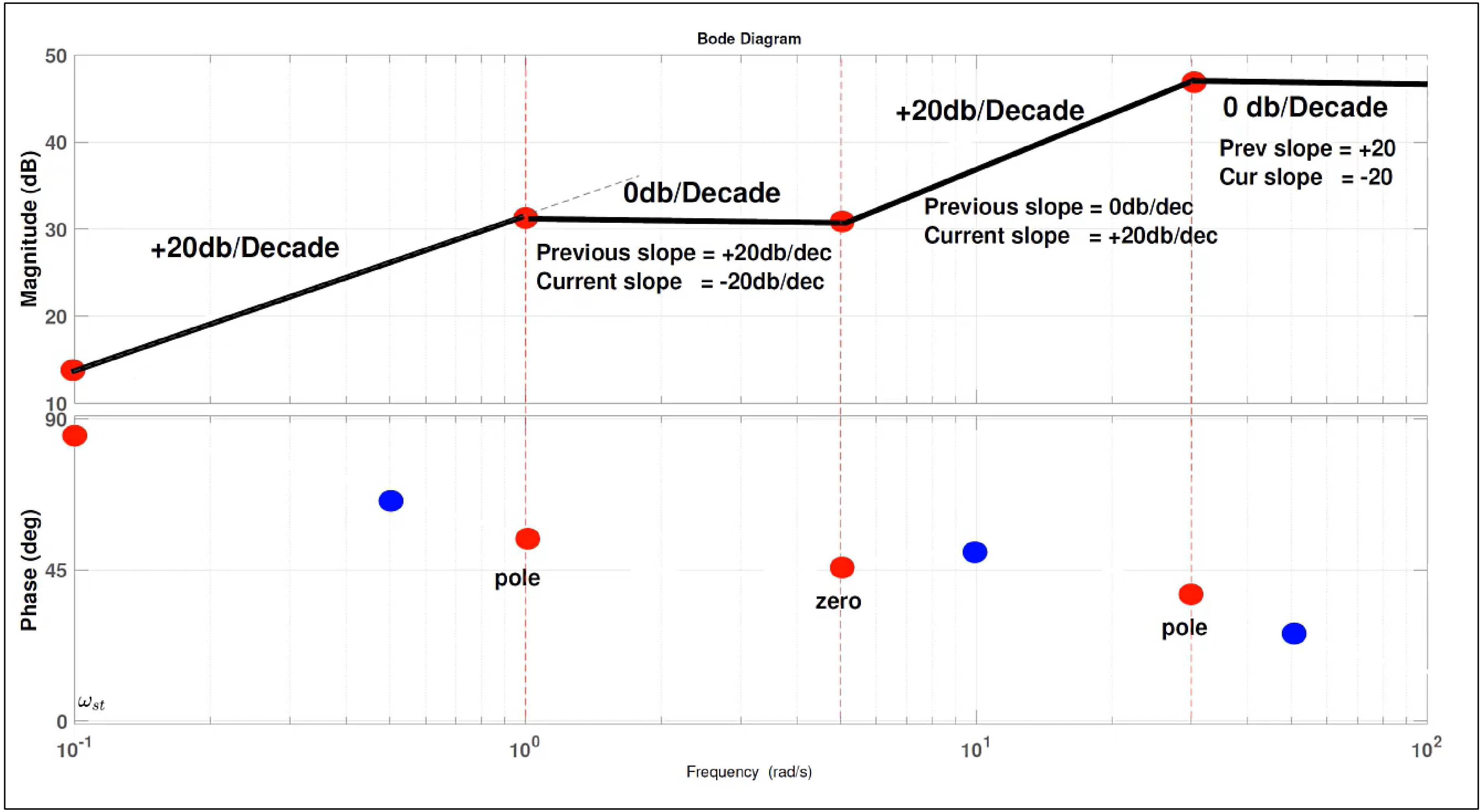
Step 5. Phase angle

$$\varphi = \tan^{-1}\left(\frac{0}{K}\right) + \underbrace{\operatorname{sgn}(k_2 - k_1)}_{-\text{ or }+}\tan^{-1}\left(\frac{\omega}{0}\right) + \tan^{-1}\left(\frac{\omega}{5}\right) - \tan^{-1}\left(\frac{\omega}{1}\right) - \tan^{-1}\left(\frac{\omega}{30}\right)$$

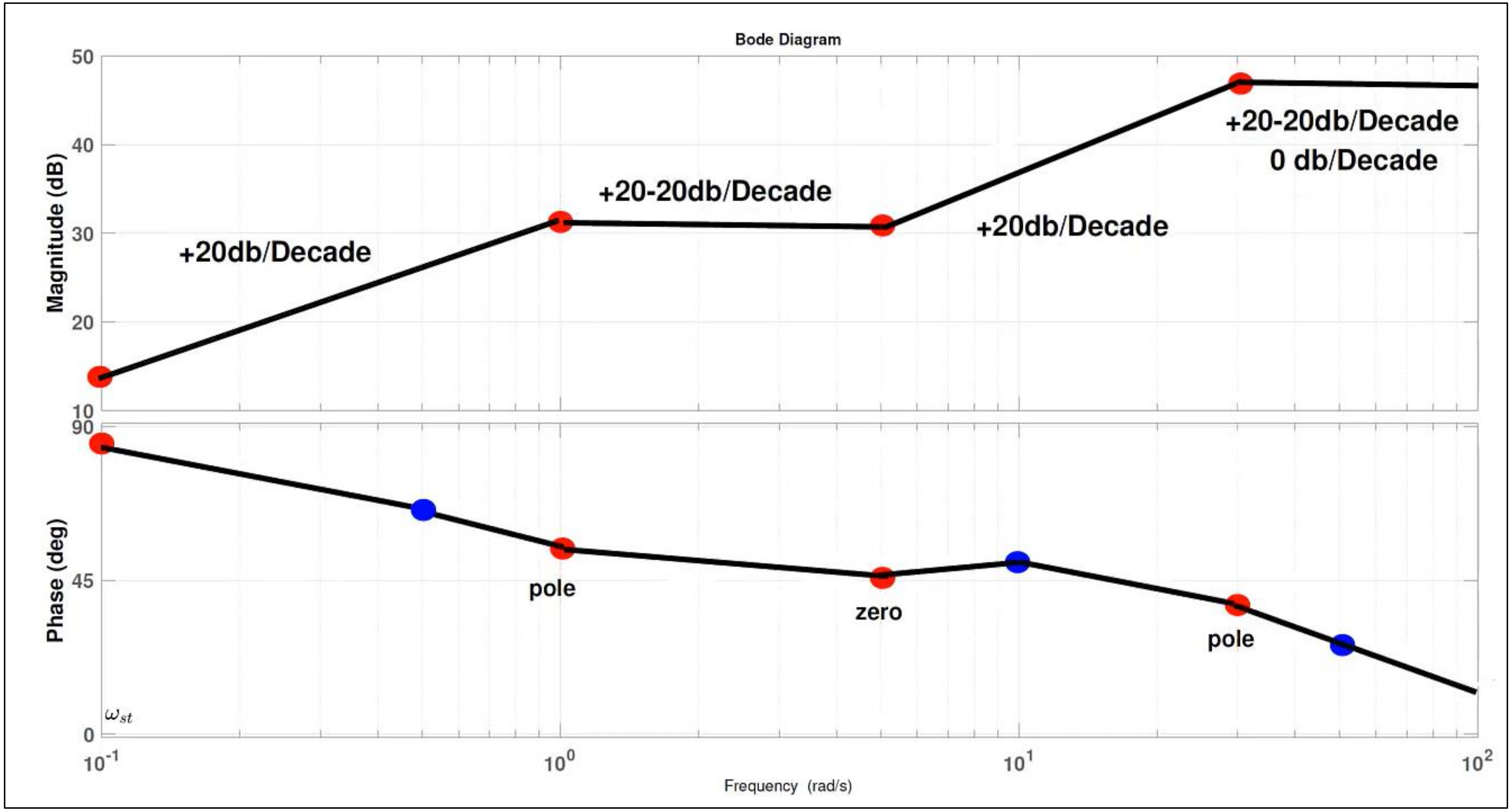
$$\varphi = 0 + 90 + \tan^{-1}\left(\frac{\omega}{5}\right) - \tan^{-1}\left(\frac{\omega}{1}\right) - \tan^{-1}\left(\frac{\omega}{30}\right)$$

ω (rad/sec)	0.1	0.5	1	5	10	30	50
φ (deg)	85.24	68.19	54.4	46.8	50.7	37.4	26.39

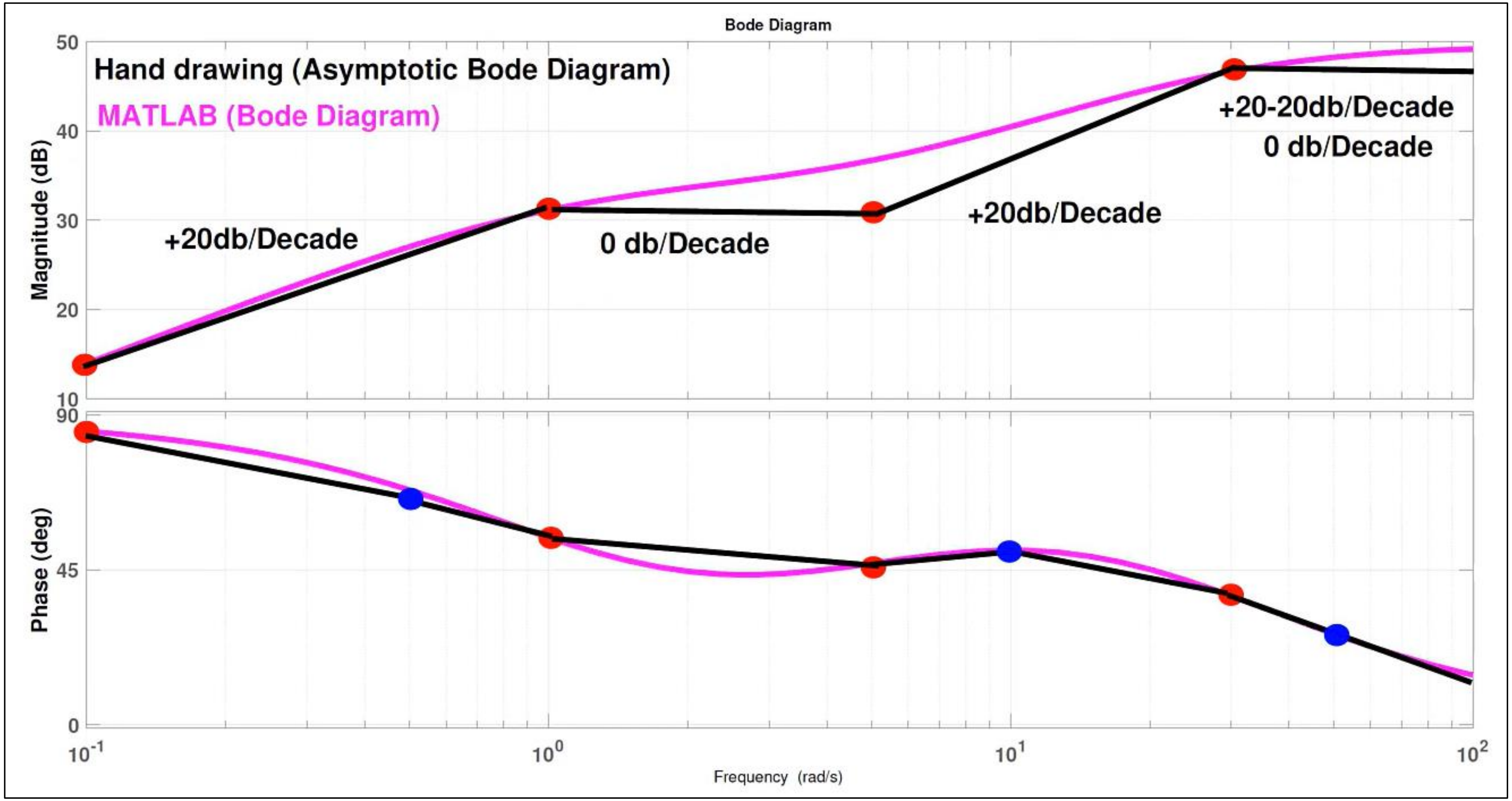
Example. 1. (Bode Diagram)



Example. 1. (Bode Diagram)



Example. 1. (Bode Diagram)



Example. 2. (Bode Diagram)

Draw the Bode plot of

$$H(s)\,G(s) = \frac{(s+2)(s+6)}{s(s+1)(s+5)(s+10)}$$

Step 1. Rewrite $H(s)\,G(s)$ and replace s by $j\omega$

$$H(j\omega)\,G(j\omega) = \frac{(j\omega+2)(j\omega+6)}{j\omega(j\omega+1)(j\omega+5)(j\omega+10)}$$

Next,

$$H(j\omega)\,G(j\omega) = \frac{2\times 6}{1\times 5\times 10} \times \frac{\left(j\frac{\omega}{2}+1\right)\left(j\frac{\omega}{6}+1\right)}{j\omega\left(j\frac{\omega}{1}+1\right)\left(j\frac{\omega}{5}+1\right)\left(j\frac{\omega}{10}+1\right)}$$

Thus,

$$H(j\omega)\,G(j\omega) = 0.24 \times \frac{\left(j\frac{\omega}{2}+1\right)\left(j\frac{\omega}{6}+1\right)}{j\omega\left(j\frac{\omega}{1}+1\right)\left(j\frac{\omega}{5}+1\right)\left(j\frac{\omega}{10}+1\right)}$$

Example. 2. (Bode Diagram)

Step 2. Draw table as below

Factor	Type	(Corner/Break) frequency	Slope
$(j\omega)^1$	pole	-	-20×1
$(j\frac{\omega}{2}+1)$	zero	2	$+20$
$(j\frac{\omega}{6}+1)$	zero	6	$+20$
$(j\frac{\omega}{1}+1)$	pole	1	-20
$(j\frac{\omega}{5}+1)$	pole	5	-20
$(j\frac{\omega}{10}+1)$	pole	10	-20

Step 3. Starting frequency

$$\omega_{st} = \frac{\text{Least value of corner frequency}}{10} = \frac{1}{10} = 0.1$$

Example. 2. (Bode Diagram)

Step 4. Starting magnitude

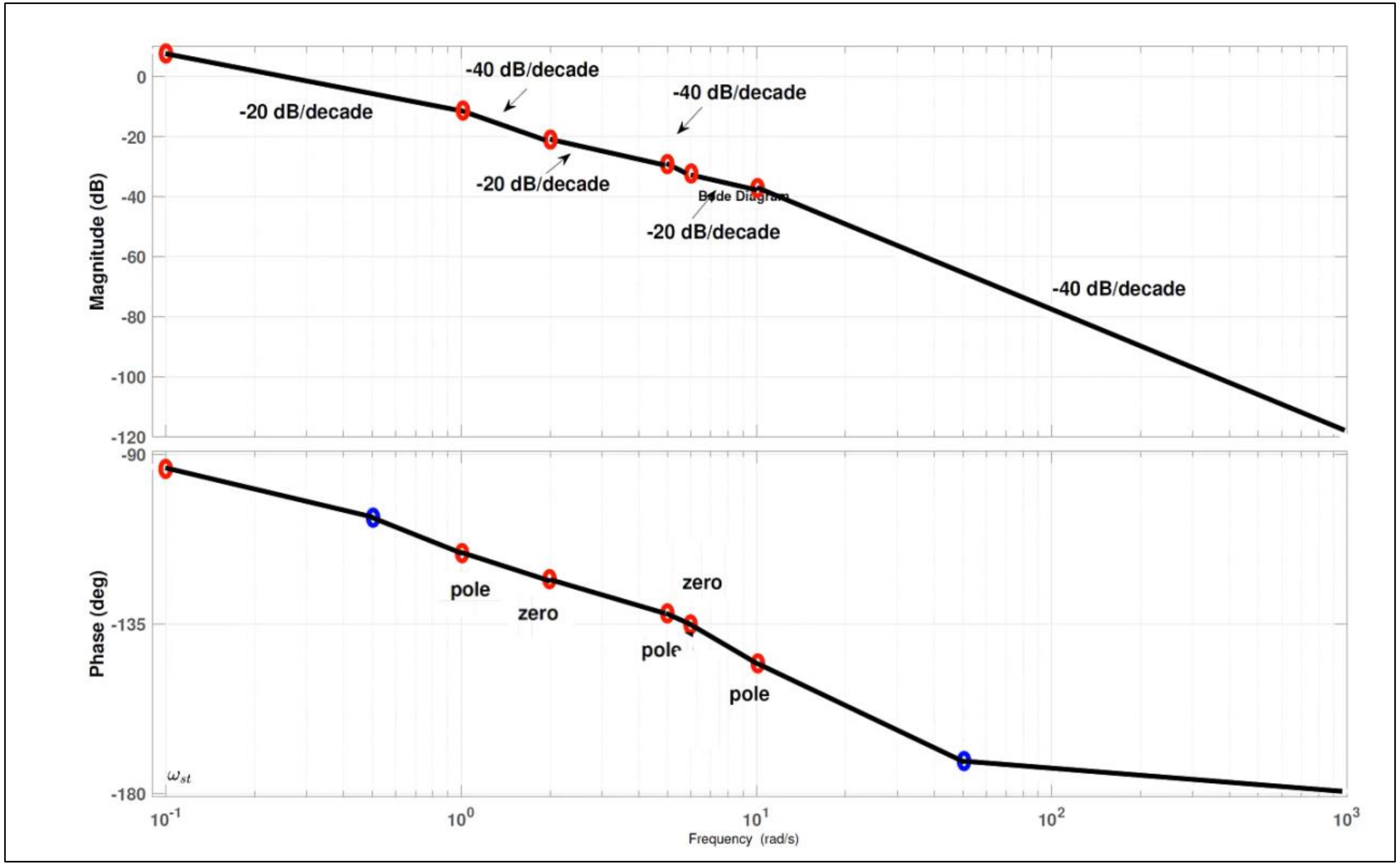
$$= 20\log(K) + 20 \times (k_2 - k_1)\log(\omega_{st})$$
$$= 20\log(0.24) - 20 \times 1 \times \log(0.1) = 7.6 \text{ dB}$$

Step 5. Phase angle

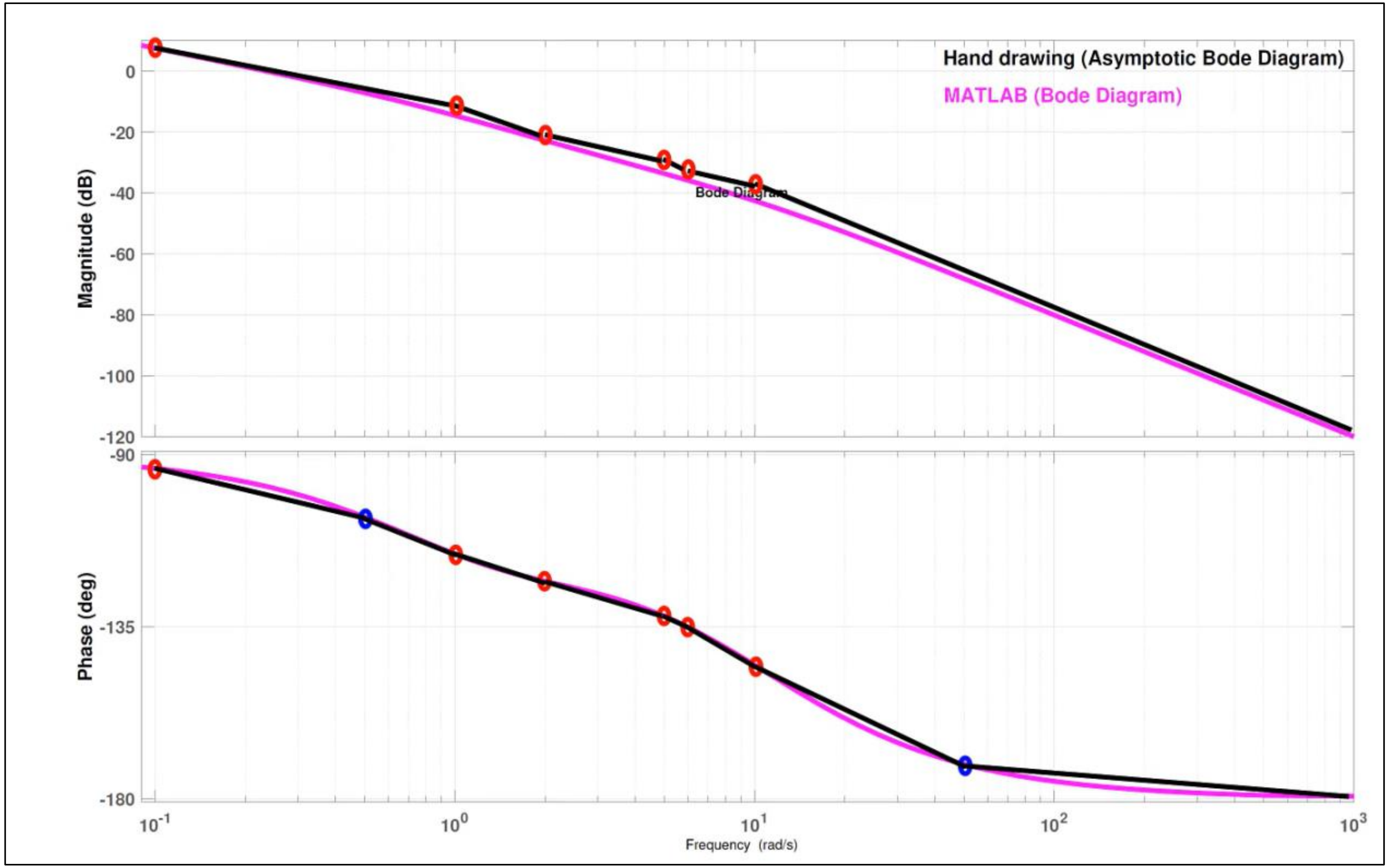
$$\varphi = -90 + \tan^{-1}\left(\frac{\omega}{2}\right) + \tan^{-1}\left(\frac{\omega}{6}\right) - \tan^{-1}\left(\frac{\omega}{1}\right) - \tan^{-1}\left(\frac{\omega}{5}\right) - \tan^{-1}\left(\frac{\omega}{10}\right)$$

ω (rad/sec)	0.1	0.5	1	2	5	6	10	50
φ (deg)	−93.6	−106	−116	−123	−132	−135	−145	−171

Example. 2. (Bode Diagram)



Example. 2. (Bode Diagram)



HOMEWORK. (Bode Diagram)

Draw the Bode plot of

$$H(s)\,G(s) = \frac{s^2 + 2s + 8}{s\,(s^2 + 2s + 10)}$$