قسم هندسة تقنيات القدرة الكهربائية

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المرحلة الثالثة

المحاضرة الثانية

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المجالات الكهرومغناطيسية

Coulomb Forces and Electric Field Intensity

Coulomb Law

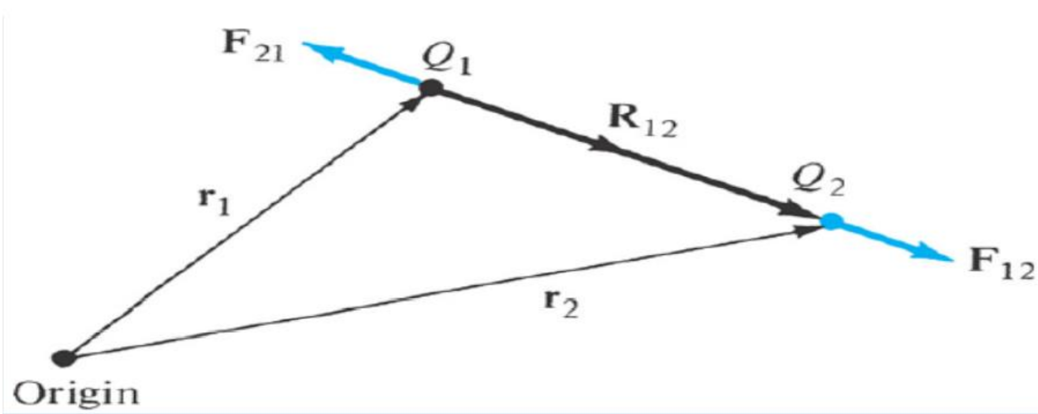
There is a force between two charges which is directly proportional to the charge magnitude and inversely proportional to the square of the separation distance.

$$F = \frac{KQ_1Q_2}{R^2} = \frac{Q_1Q_2}{4\pi\varepsilon_0 R^2}$$

The force is in newtons (N), the distance is in meters (m), and the (derived) unit of charge is the coulomb (C). ε_0 is the permittivity of free space farads per meter (F/m),

$\varepsilon_0 = 8.85 \times 10^{-12} \cong \frac{10^{-9}}{36\pi} F/m$

$k(constant) \rightarrow K = \frac{1}{4\pi\varepsilon_0} \cong 9 \times 10^{-9}\ m/F$



→ The force F_{12} on Q_2 due to Q_1 is:

$$\vec{F}_{12} = \frac{Q_1Q_2}{4\pi\varepsilon_0 R^2} a_{R_{12}} = \frac{Q_1Q_2(r_2 - r_1)}{4\pi\varepsilon_0 |r_2 - r_1|^3}$$

$$where\ a_{R_{12}} = \frac{\vec{R}_{12}}{R} = \frac{r_2 - r_1}{R}\ ,\ R = |\vec{R}_{12}|$$

→ The force F_{12} on Q_2 due to Q_1 is:

$$\vec{F}_{21} = |F_{12}| a_{R_{21}} = |F_{12}|(-a_{R_{12}})$$

$$or \quad \vec{F}_{21} = -\vec{F}_{12}$$

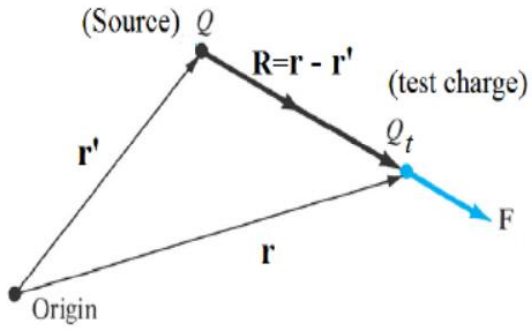
$aR_{12} = -aR_{21}$

Electric Field Intensity (E):

The electric field intensity (or electric field strength) E is the force per unit charge when placed in an electric field.

$$E = \frac{F}{Q_t}$$

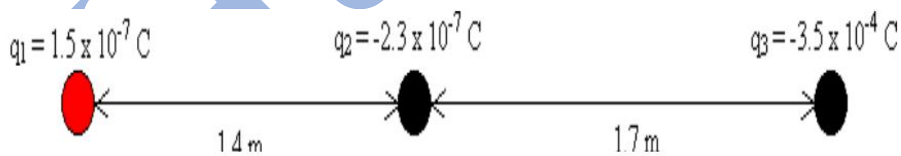
$$F = \frac{Q\,Q_t(r - r')}{4\pi\varepsilon_0 |r - r'|^3}$$



$$\rightarrow E = \frac{F}{Q_t} = \frac{Q\,(r - r')}{4\pi\varepsilon_0 |r - r'|^3} = \frac{Q}{4\pi\varepsilon_0 R^2} a_R \quad \text{(Newton/Coulomb) or (V/m)}$$

E: Electric field intensity at point r due to a point charge (Q)

Example 1: The following three charges are arranged as shown below. Determine the net force acting on the charge on the far right (q3):



$$F_e = \frac{k\,q_1 q_3}{r^2} = \frac{8.99e9(1.5e\text{-}7)(3.5e\text{-}4)}{3.1^2} = 0.049112903\ N$$

$$F_e = \frac{k\,q_2 q_3}{r^2} = \frac{8.99e9(2.3e\text{-}7)(3.5e\text{-}4)}{1.7^2} = 0.250413495\ N$$

F_{NET} = - 0.049112903 N + +0.250413495 N = 0.201300592 = 0.20 N

Example 2: A point charge $Q_1 = 2\ mC$ is located at $p_1(-3,7,-4)$ while $Q_2 = 5mC$ at $p_1(2,4,-1)$. Find F_{12} and F_{21}.

Sol: $\overrightarrow{R_{21}} = [2-(-3)]a_x + [4-7]a_y + [-1-(-4)]a_z$

$$= 5a_x - 3a_y + 3a_z$$

$$|R_{12}| = \sqrt{5^2 + (-3)^2 + 3^2} = \sqrt{43}$$

$$R_{12}^2 = 43$$

$$aR_{12} = \frac{5a_x - 3a_y + 3a_z}{\sqrt{43}}$$

$$F_{12} = \frac{Q_1 Q_2}{4\pi\varepsilon_0 R_{12}^2} aR_{12}$$

$$= \frac{2 \times 10^{-3} \times 5 \times 10^{-3}}{4\pi \times \frac{10^{-9}}{36\pi} \times 43} \times \frac{5a_x - 3a_y + 3a_z}{\sqrt{43}}$$

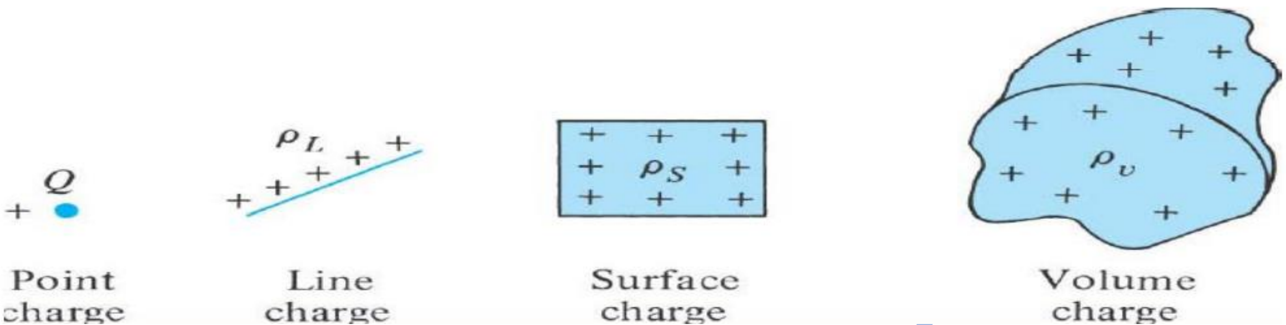
$$= 1594.7a_x - 957.2a_y + 957.2a_z$$

$$F_{21} = -F_{12}$$

$$= -1594.7a_x + 957.2a_y - 957.2a_z$$

HW: Point charges $1mC$ and $-2\ mC$ are located at $(3,2,-1)$ and $(-1,-1,4)$, respectively. Calculated the electric force on a $10\ nC$ charge located at $(0,3,1)$ and the electric field intensity at that point.

CHARGE DISTRIBUTIONS:



To find the charge element dQ:

(Line Charge)

$$dQ = \rho_L dl \rightarrow Q = \int \rho_L dl$$

$$dQ = \rho_s dS \rightarrow Q = \int_S \rho_s dS$$

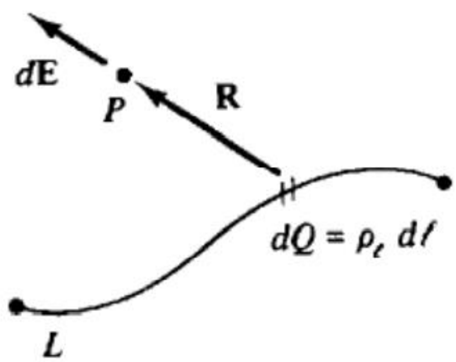
(Surface Charge)

$$dQ = \rho_v dV \rightarrow Q = \int_V \rho_v dV$$

(Volume Charge)

Line Charge:

If charge is distributed over a (curved) line, each differential charge dQ along the line produces a differential electric field



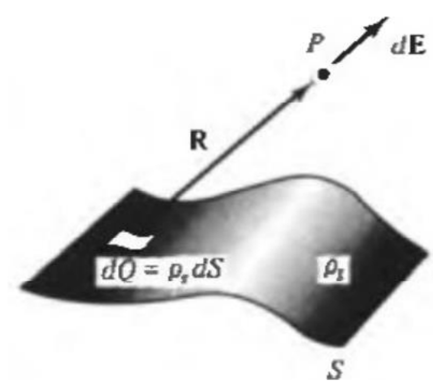
$$dE = \frac{dQ}{4\pi\varepsilon_0 R^2} a_R$$

At p in the figure above. And if line charge density $\boldsymbol{\rho_L}$ (in C/m), and no other charge is in the region, then the total electric field at p is:

$$E = \int_L \frac{\rho_L dl}{4\pi\varepsilon_0 R^2} a_R$$

Surface (sheet) Charge:

Charge may also be distributed over a surface or a sheet. Then each differential charge dQ on the sheet results in a differential field

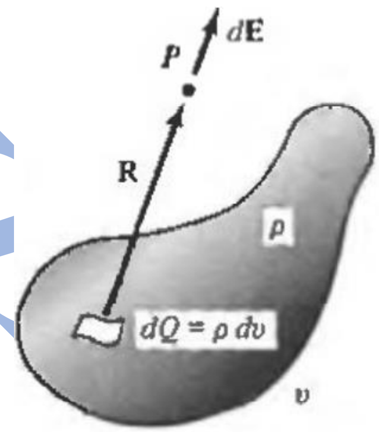


$$dE = \frac{dQ}{4\pi\varepsilon_0 R^2} a_R$$

At p in the figure above. And if **surface charge density $\boldsymbol{\rho_s}$** (in C/m^2), and no other charge is in the region, then the total electric field at p is:

$$E = \int_S \frac{\rho_S dS}{4\pi\varepsilon_0 R^2} a_R$$

Volume charge:



When charge is distributed throughout a specified volume, each charge element contributes to the electric field at an external point. With reference to volume v in figure shown, each differential charge dQ produces a differential electric field:

$$dE = \frac{dQ}{4\pi\varepsilon_0 R^2} a_R$$

At the observation point P. Assuming that the only charge in the region is contained within the volume, the total electric field at P is obtained by: