Bilad Alrafidain University College

Electric Power Techniques Engineering Department

Control Systems Analysis

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Control Systems Analysis

Course Contents

- Introduction to Control System.
- Transfer Function.
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Lecture Two

Transfer Function

Introduction

The Transfer Function (T.F) is a convenient representation of a linear time invariant dynamical system. Mathematically the transfer function is a function of complex variables. For finite dimensional systems the transfer function is simply a rational function of a complex variable. The transfer function can be obtained by inspection or by simple algebraic manipulations of the differential equations that describe the systems. Transfer functions can describe systems of very high order, even infinite dimensional systems governed by partial differential equations. The transfer function of a system can be determined from experiments on a system.

The Transfer Function (T.F) it is the ratio of Laplace transform of output signal to Laplace transform of input signal assuming all the initial conditions to be zero.

Let, there is a given system with input r(t) and output c(t) as shown in (Figure 1), then its Laplace domain is shown in (Figure 2). Here, input and output are R(s) and C(s) respectively.

Figure 1 : The left side is a system in time domain – The right side is the system in frequency domain.

G (S) is the **Transfer Function (T.F) :** $G(S) = \frac{C\ (S)}{R\ (S)}$ where all the initial conditions are zero.

Transfer Function

The applicability of the concept of the transfer function is limited to linear, time-invariant, differential equation systems. The transfer function approach, however, is extensively used in the analysis and design of such systems. In what follows, we shall list important comments concerning the transfer function.

1. The transfer function of a system is a mathematical model in that it is an operational method of expressing the differential equation that relates the output variable to the input variable.
2. The transfer function is a property of a system itself, independent of the magnitude and nature of the input or driving function.
3. The transfer function includes the units necessary to relate the input to the output; however, it does not provide any information concerning the physical structure of the system. (The transfer functions of many physically different systems can be identical.)
4. If the transfer function of a system is known, the output or response can be studied for various forms of inputs with a view toward understanding the nature of the system.
5. If the transfer function of a system is unknown, it may be established experimentally by introducing known inputs and studying the output of the system. Once established, a transfer function gives a full description of the dynamic characteristics of the system, as distinct from its physical description.

Example 1 : Determine the Transfer Function shown in (Figure 2) ?

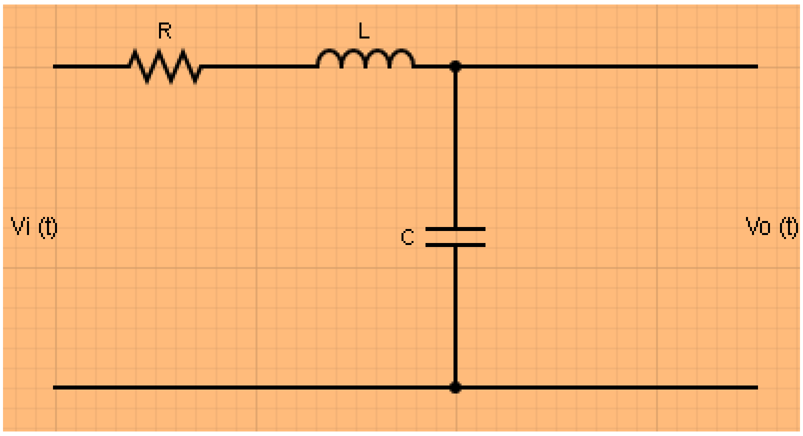

Figure 2 : A system in time domain

Solution : First, redrawn the system shown in (Figure 2) in frequency domain as shown in (Figure 3) below:

Taking Laplace Transform to the components of (Figure 2).

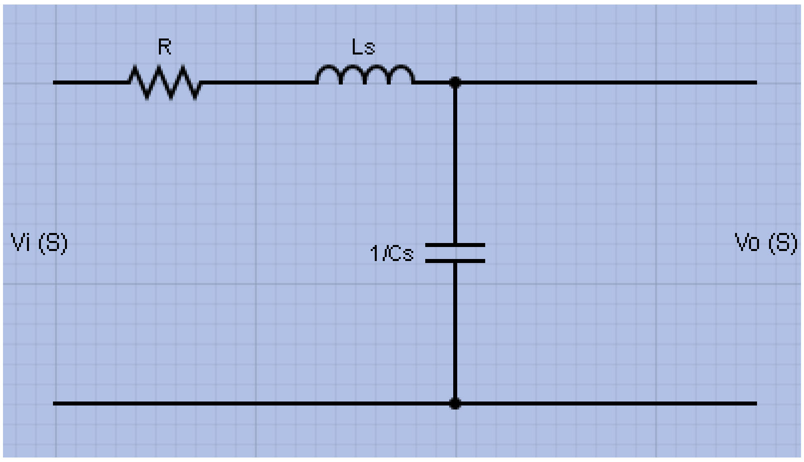

Figure 3 : A system in frequency domain

Applying KVL to loop-1 of (Figure 3):

$$V_i(S) = \left(R + Ls + \frac{1}{Cs}\right) I(S) \ldots (1)$$

Applying KVL to loop-2 of (Figure 3):

$$V_o(S) = \left(\frac{1}{Cs}\right) I(S) \ldots (2)$$

From equation (2) $I(S)$ can be obtained as :

$$I(s) = \frac{V_o(S)}{1/_{Cs}} = V_o(S)Cs \ldots (3)$$

Now, Substitute equation (3) in equation (1):

$$V_i(S) = \left(R + Ls + \frac{1}{Cs}\right) V_o(S)Cs$$

$$\frac{V_o(S)}{V_i(S)} = \frac{1}{\left(R + Ls + \frac{1}{Cs}\right) Cs} = \frac{1}{(1 + RCs + LCs^2)}$$

Then, the Transfer Function of the given system is:

$$G(S) = \frac{1}{(1 + RCs + LCs^2)}$$

Example 2 : Determine the Transfer Function shown in (Figure 4) ?

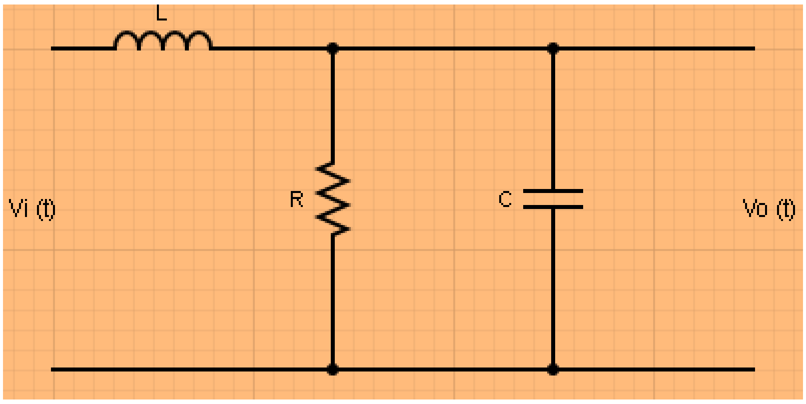

Figure 4 : A system in time domain

Solution : First, redrawn the system shown in (Figure 4) in frequency domain as shown in (Figure 5) below:

Taking Laplace Transform to the components of (Figure4).

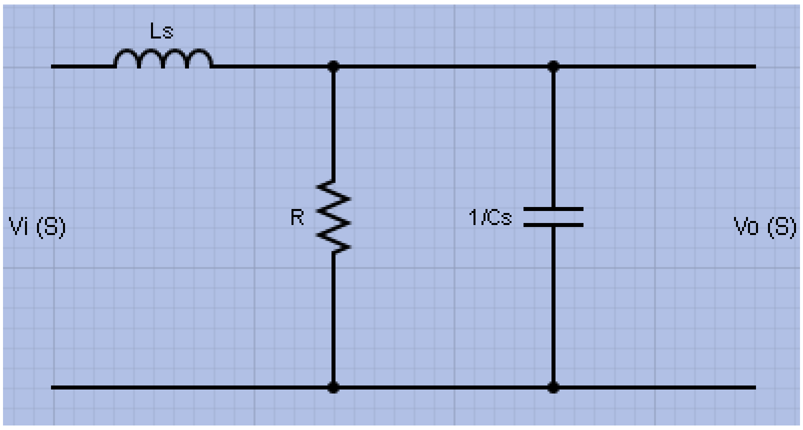

Figure 5 : A system in frequency domain

Applying KVL to loop-1 of (Figure 5):

$$V_i(S) = \; Ls\, I(S) + R\, I(S) \ldots (1)$$

Applying KVL to loop-2 of (Figure 5):

$$V_o(S) = \; V_C(S) = V_R(S) \ldots (2)$$

$$I\,(S) = I_1(S) + I_2(S)$$

$$I_1(S) = \frac{V_O(S)}{R}$$

$$I_2(S) = \frac{V_O(S)}{1/Cs}$$

Now, Substitute equation (2) in equation (1):

$$V_i(S) = V_O(S) + Ls\,I(S)$$

$$V_i(S) = V_O(S) + Ls\,(I_1(S) + I_2(S))$$

$$V_i(S) = V_O(S) + Ls\left(\frac{V_O(S)}{R} + \frac{V_O(S)}{1/Cs}\right) \dots (3)$$

From equation (3) we take $V_O(S)$ as a common factor:

$$V_i(S) = V_O(S)\left(1 + Ls\left(\frac{1}{R} + Cs\right)\right)$$

$$V_i(S) = V_O(S)\left(1 + \frac{Ls}{R} + LCs^2\right)$$

$$\frac{V_O(S)}{V_i(S)} = \frac{1}{1 + \frac{Ls}{R} + LCs^2}$$

Then, the Transfer Function of the given system is:

$$G(S) = \frac{1}{1 + \frac{Ls}{R} + LCs^2}$$