Bilad Alrafidain University College

Electric Power Techniques Engineering Department

Control Systems Analysis

Fourth Stage

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Lecture Sixteen

Frequency-Response Approach to Control System Design

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Frequency-Response Approach to Control System Design

The Desired Performance Interims of Frequency Response (in Bode Diagram).

The desired frequency response performance of the systems are generally given interims of

a) PM; $30^{\circ} \leq PM \leq 60^{\circ}$ and GM; $GM \geq 6$ dB

b) The static error constants Kp, Kv, Ka.

c) At the gain crossover frequency, ω_{gcf}, the slope of the log-magnitude curve in the Bode diagram is -20 dB/decade,

d) The system band wide BW frequency has to be sufficiently large enough to contain system operation frequency.

Frequency-Response Approach to Control System Design

Basic Characteristics of Lead, Lag, and Lag-Lead Compensation.

Lead compensation essentially yields an appreciable improvement in transient response and a small change in steady-state accuracy. It may accentuate high-frequency noise effects.

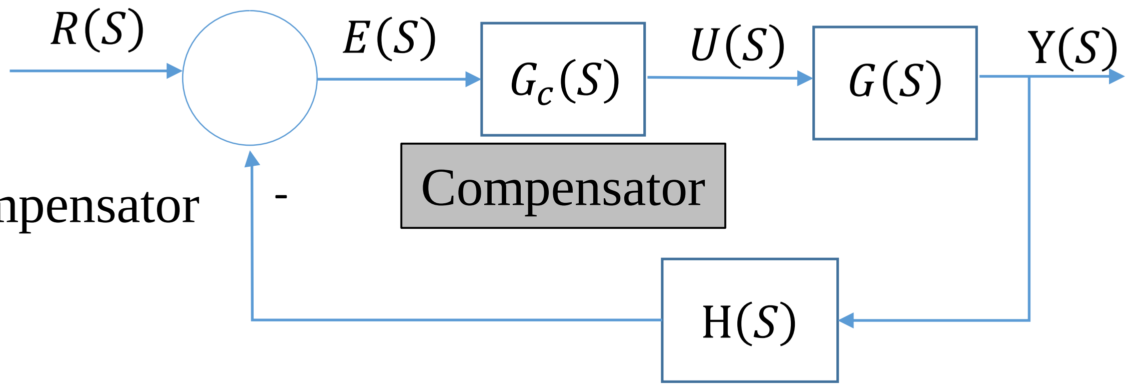
Lag compensation yields an appreciable improvement in steady-state accuracy at the expense of increasing the transient-response time. Lag compensation will suppress the effects of high-frequency noise signals.

Lag-lead compensation combines the characteristics of both lead compensation and lag compensation.

Frequency-Response Characteristics of the Lead Compensator.

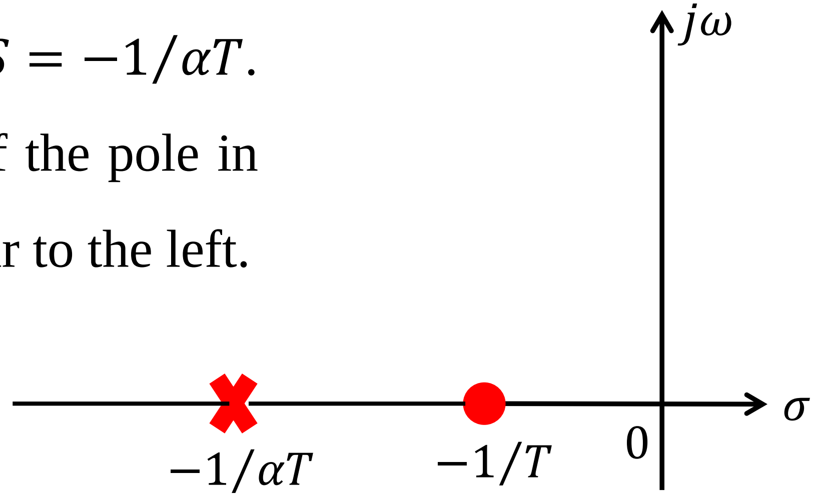
Consider a lead compensator having the following transfer function:

$$\frac{U(S)}{E(S)} = K_c\alpha\,\frac{Ts+1}{\alpha Ts+1} = K_c\,\frac{s+1/T}{s+1/\alpha T}\,,(0<\alpha<1)$$



where α is called the attenuation factor of the lead compensator and $(K_c, \alpha\ and\ T)$ are design parameters.

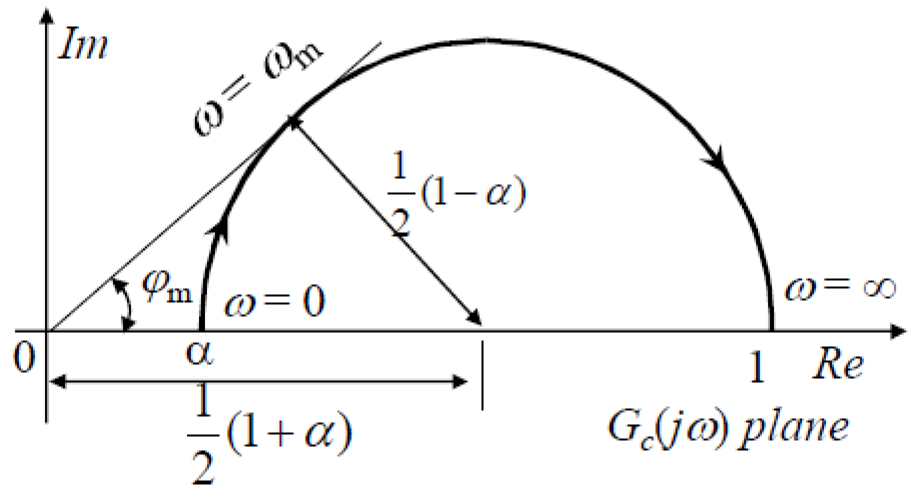
The phase lead compensator has a zero at $S=-1/T$ and a pole at $S=-1/\alpha T$. Since $0<\alpha<1$, we see that the zero is always located to the right of the pole in the complex plane. Note that for a small value of α the pole is located far to the left.



Frequency-Response Characteristics of the Lead Compensator.

Polar plot of a lead compensator: For $K_c = 1$ Polar plot of a lead compensator is

$$G_c(j\omega) = \alpha \frac{Tj\omega + 1}{\alpha Tj\omega + 1} = \frac{j\omega + 1/T}{j\omega + 1/\alpha T}, (0 < \alpha < 1)$$



For a given value of α, at frequency $\omega = \omega_m$ the angle between the positive real axis and the tangent line drawn from the origin to the semicircle gives the maximum phase-lead angle, $\emptyset_m$.

$$\sin \emptyset_m = \frac{1 - \alpha}{1 + \alpha}$$

Frequency-Response Characteristics of the Lead Compensator.

The design steps for phase lead compensator.

Step 1: To satisfy the requirement on the given static error constant determine gain K_T;

a) with the system’s variable gain K by letting $K_c = 1$ and $K_T = K\alpha$, if the system’s gain is constant then

b) with $K_T = K_c\alpha$.

Step 2: Using the gain K_T, draw the Bode diagram of $K_T G(j\omega)H(j\omega)$ that the gain adjusted but uncompensated system. Evaluate the phase margin.

Step 3: Determine the necessary phase-lead angle to be added to the system. Add an additional phase 50 to 120 to the phase-lead angle required, because the addition of the lead compensator shifts the gain crossover frequency to the right and decreases the phase margin.

Frequency-Response Characteristics of the Lead Compensator.

Step 4: Determine the frequency where the magnitude of the uncompensated system $K_T G(j\omega)H(j\omega)$ is equal to $-20\log(1/\sqrt{\alpha})$. Select this frequency as the new gain crossover frequency (ω_{ngc}) . This frequency corresponds to $\omega_{ngc} = \omega_m = (1/T\sqrt{\alpha})$ where the maximum phase shift $\emptyset_m$ occurs.

Determining $\omega_{ngc} = \omega_m$ where $|K_T G(j\omega)H(j\omega)| = -20\log(1/\sqrt{\alpha})$.

a) Analytically by:

$$\left|G_1(j\omega)\right|\Big|_{\omega=?} = -20\log\frac{1}{\sqrt{\alpha}} = 20\log\sqrt{\alpha} \quad dB$$

Since K_T and α are determined then,

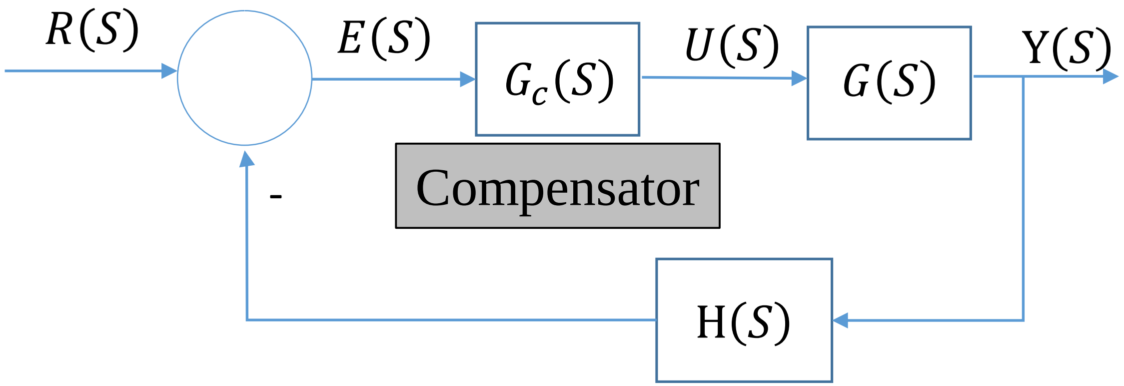
a) calculate constant k: $K = \frac{K_T}{\alpha}$ if the system has variable gain k

b) calculate constant K_c from: $K_c = \frac{K_T}{\alpha}$ if the system has a fixed gain

Frequency-Response Characteristics of the Lead Compensator.

Example 1: Consider the system shown in Figure where the transfer function of blocks are;

$$G(S) = \frac{4}{S(S+2)}, H(S) = 1$$



The desired frequency response performance: It is desired to design a compensator $G_c(S)$ for the system so that the static velocity error constant K_v is 20 sec^{-1}, the phase margin is at least 50°,($PM_d \geq 50^{\circ}$) and the gain margin is at least 10 dB, ($GM_d \geq 10\ dB$).

Frequency-Response Characteristics of the Lead Compensator.

Solution:

Step 1: Since the system has fixed gain them $K_T = K_c\alpha$. To adjust the gain K_T to meet the steady-state performance specification or to provide the required static velocity error constant. Since the static velocity error constant K_v is 20 sec^{-1}, we obtain;

$$K_v = \lim_{S\to 0} SG_C(S)G(S) = \lim_{S\to 0} S\,\frac{TS+1}{\alpha TS+1} * \frac{4K_T}{S(S+2)} \qquad 20 = 2K_T \;\rightarrow K_T = 10$$

Step 2: Draw a Bode diagram of $K_T G(j\omega)H(j\omega)$ $\quad K_T G(j\omega)H(j\omega) = \frac{40}{j\omega(j\omega+2)}$

From the bode plot in (Step 2), the phase margin is equal to 17°, while its desired to be at least 50°,$(PM_d \geq 50^{\circ})$. $\emptyset_m = 50^{\circ} - 17^{\circ} = 33^{\circ}$add a little bit, to compensate the possible phase shift in the new gain cross over frequency. $\emptyset_m = (50^{\circ} - 17^{\circ}) + 5 = 38^{\circ}$

Frequency-Response Characteristics of the Lead Compensator.

Step 3: Determine design parameter α; for $\emptyset_m = 38^{\circ}$ we have

$$\sin\emptyset_m = \frac{1-\alpha}{1+\alpha} \rightarrow \alpha = \frac{1-sin\emptyset_m}{1+sin\emptyset_m} = \frac{0.384}{1.615} = \alpha = 0.24$$

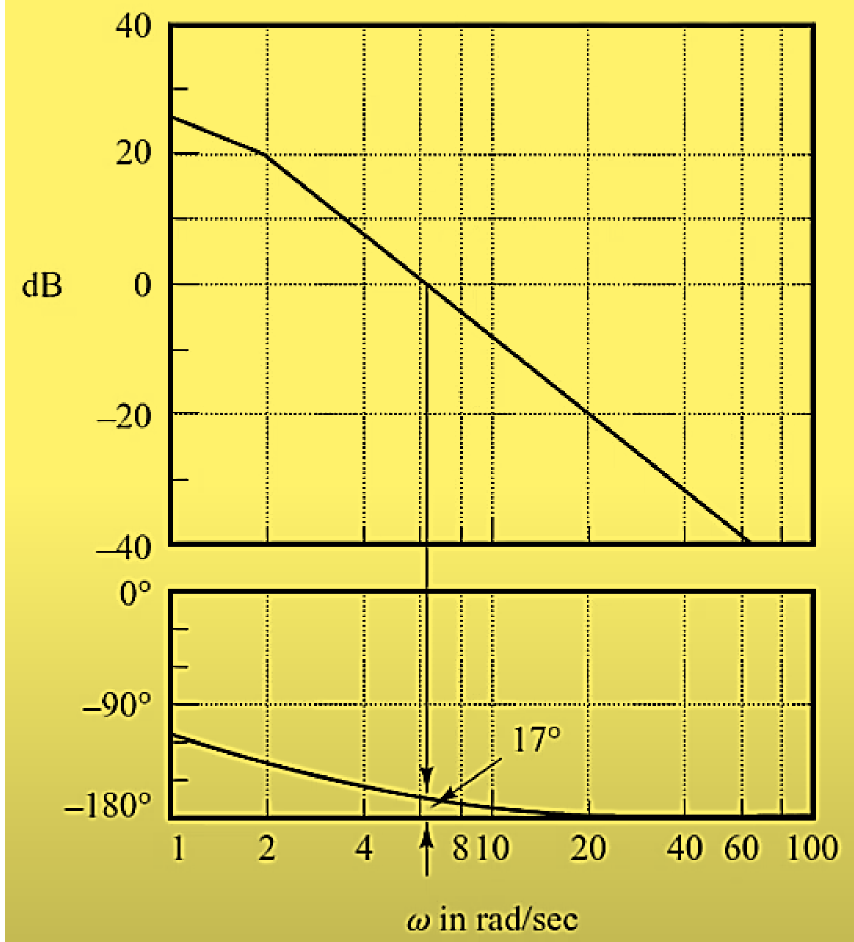
$$\alpha = 0.24 \ \rightarrow \ K_c = \frac{K_T}{\alpha} = \frac{10}{0.24} = 41.66$$

Step 4: Determine the frequency where the magnitude of the uncompensated system $|K_T G(j\omega)H(j\omega)|$ is equal to $-20\log(1/\sqrt{\alpha})$.

Determining $\omega_{ngc} = \omega_m$ where $|K_T G(j\omega)H(j\omega)| = -20\log(1/\sqrt{\alpha})$

$$K_T G(j\omega)H(j\omega) = \frac{40}{j\omega(j\omega+2)} \qquad \omega_m = \frac{1}{\sqrt{\alpha}T}$$

$|K_T G(j\omega)H(j\omega)| = -20\log(1/\sqrt{\alpha})$ When $\omega = \omega_{ngc}$



Frequency-Response Characteristics of the Lead Compensator.

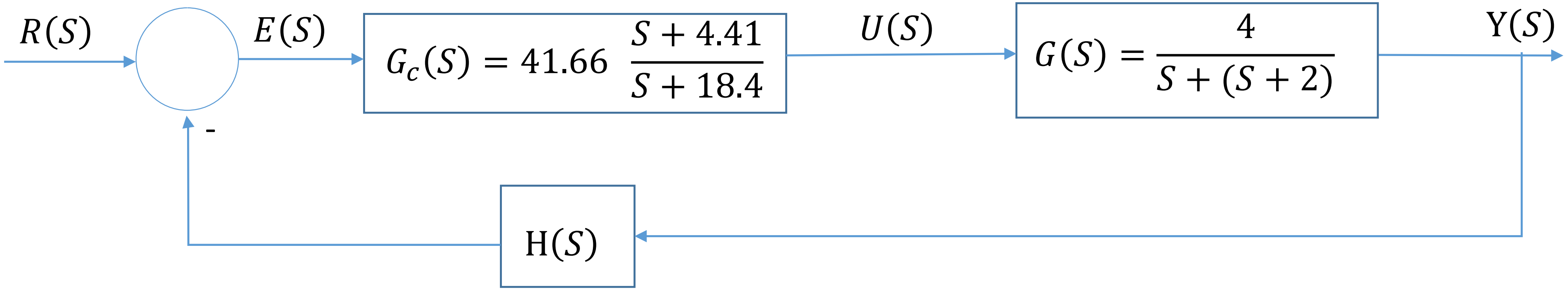
$$\left|\frac{40}{j\omega_{ngc}(j\omega_{ngc}+2)}\right| = -20\ \log(1/\sqrt{0.24}) \qquad \left|\frac{40}{j\omega_{ngc}(j\omega_{ngc}+2)}\right| = -6.2\ dB \rightarrow \omega_{ngc} \approx 9\ rad/sec$$

$$\omega_{ngc} = \omega_m = \frac{1}{\sqrt{\alpha T}} \rightarrow 9 = \frac{1}{\sqrt{0.24 T}} \rightarrow T = 0.2268$$

Now we have the following parameters:

$$K_T = 10, \alpha = 0.24, K_c = 41.66\ and\ T = 0.2268 \qquad G_c(S) = K_c\frac{S+1/T}{S+1/\alpha T} \rightarrow G_c(S) = 41.66\ \frac{S+4.41}{S+18.4}$$

The compensated system is given by:



Frequency-Response Characteristics of the Lead Compensator.

The effect of the lead compensator is:

- Phase margin: from 17° to 50° which is better transient response with less overshoot.
- ω_{ngc}: from 6.3rad/sec to 9 rad/sec so that the system response is faster.
- Gain margin remains ∞.
- K_v is 20, as required acceptable steady-state response.

Bode diagram for the compensated system

$$G_c(S) = 41.66 \ \frac{S + 4.41}{S + 18.4}$$