Bilad Alrafidain University College

Electric Power Techniques Engineering Department

Control Systems Analysis

Fourth Stage

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Lecture Fourteen

Frequency Domain Analysis

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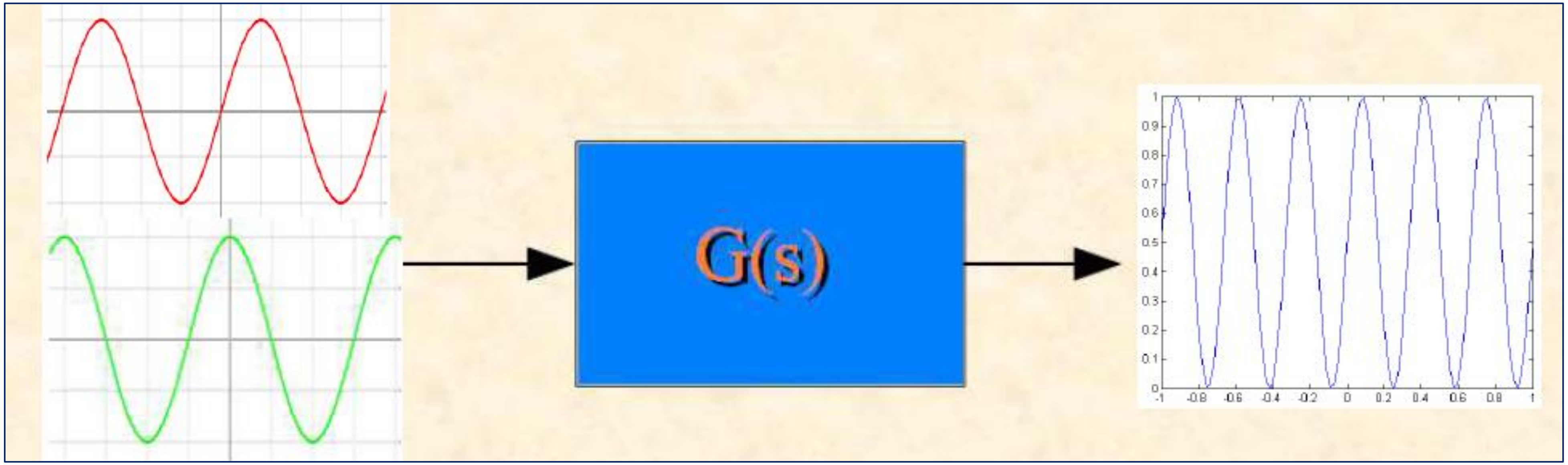
Frequency Domain Analysis

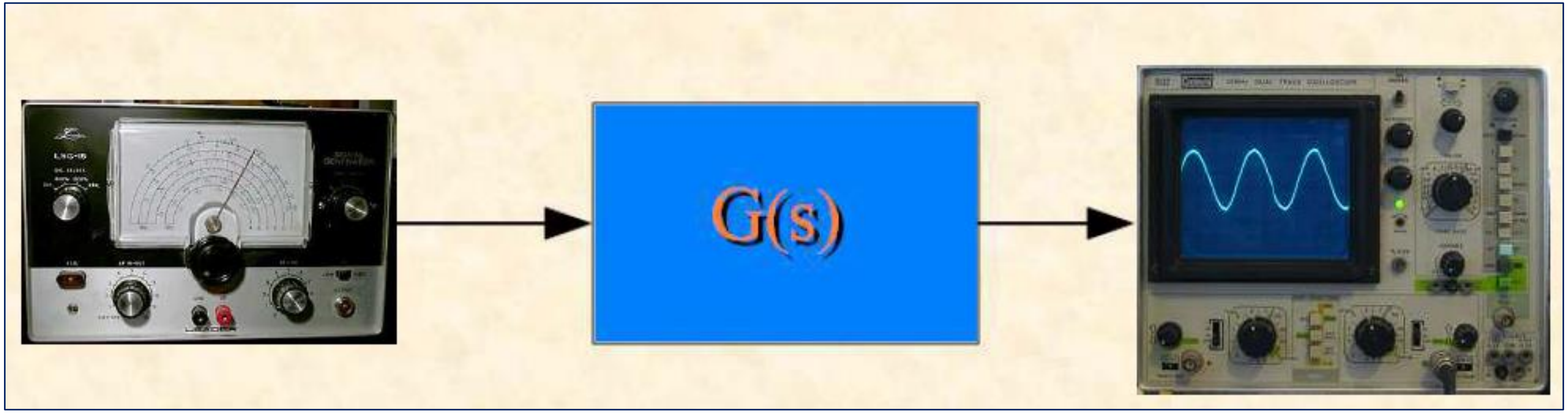
The frequency response is the output of the system in steady state when the input of the system is sinusoidal

Methods of *system analysis* by the frequency response, as the "Bode Diagrams" or the "Nyquist Plot", are the most conventional techniques used in Engineering for analysis and project of Control Systems



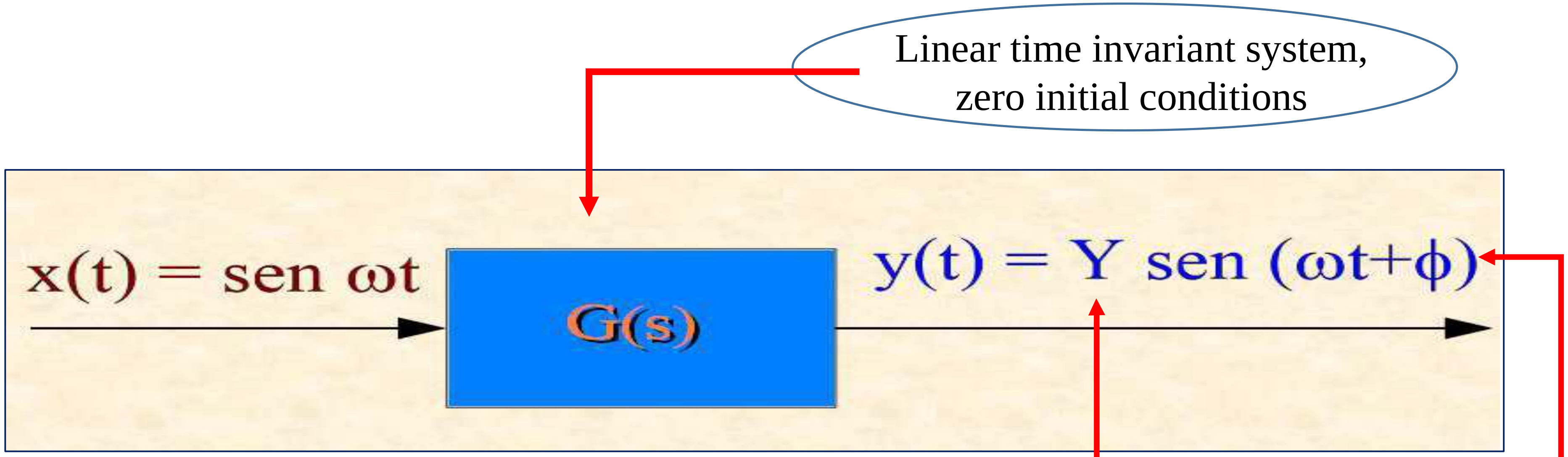
Frequency Domain Analysis

The advantage of these methods of analysis of systems by the frequency response is that they allow us to find both the *absolute* and *relative* stability of linear systems in *closed loop* only with the knowledge of frequency response in *open loop*, which can be experimentally obtained with signal generators (*sinusoidal*) and precision measurements instruments (*both easily available in laboratory*)



Frequency Domain Analysis

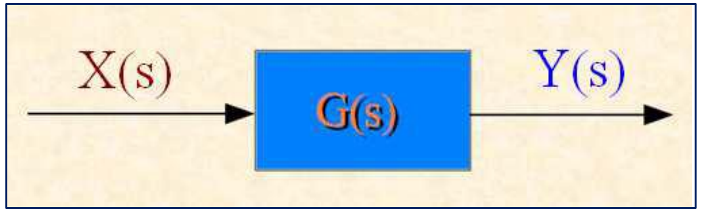
Therefore, the analysis of complicated systems can be done through tests of *frequency response* without being necessary to determine the *roots* of the characteristic equation (i.e., the *poles* of the *system*)



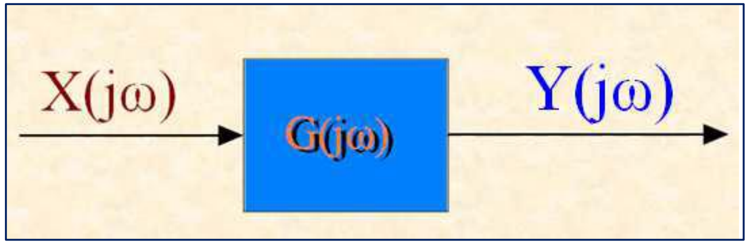
The output y(t) will have the same *frequency* of the input x(t), but, the amplitude *Y* and the phase angle ϕ, will be, in general, *different*.

Frequency Domain Analysis

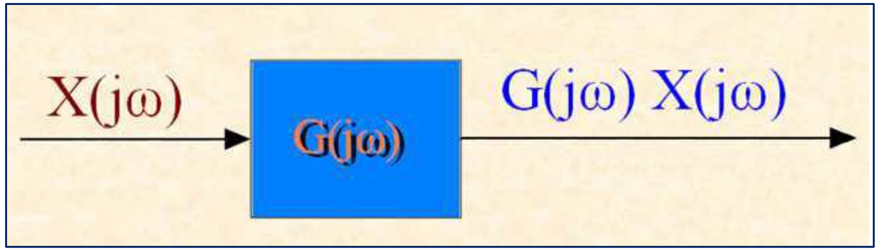
As a matter of fact, we have



And making $S = 0 + j\omega$



that is,



Frequency Domain Analysis

That is, making $S = 0 + j\omega$ in the transfer function G(s), one obtain G($j\omega$)

$$G(j\omega) = |G(j\omega)|.\, e^{j\phi}$$

Absolute value of $G(j\omega)$

Phase of $G(j\omega)$

where the phase

$$\phi = \angle G(j\omega) = \tan^{-1}\frac{Im\,(G(j\omega))}{Re\,(G(j\omega))}$$

Frequency Domain Analysis

The function $G(j\omega)$ is called the sinusoidal transfer function.

A stable, linear, time-invariant system subjected to a sinusoidal input will, at steady state, have a sinusoidal output of the same frequency as the input. But the amplitude and phase of the output will, in general, be different from those of the input.

Frequency Domain Analysis

$$|G(j\omega)| = \frac{|Y(j\omega)|}{|X(j\omega)|} = \frac{Y}{X}$$

{ ratio between the *output* and the *input*'s amplitude

$$\phi = \angle G(j\omega) = \angle Y(j\omega) - \angle X(j\omega)$$

{ difference between the phase angle of the *output* and the *input*

Frequency Domain Analysis

where the phase

$$\phi = \angle G(j\omega) = \begin{cases} < 0 & \text{phase delay} \\ = 0 & \text{in phase} \\ > 0 & \text{phase in advance} \end{cases}$$

being

$$-\pi < \phi < \pi$$