Bilad Alrafidain University College

Electric Power Techniques Engineering Department

Control Systems Analysis

Fourth Stage

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Lecture Thirteen

Root Locus Method

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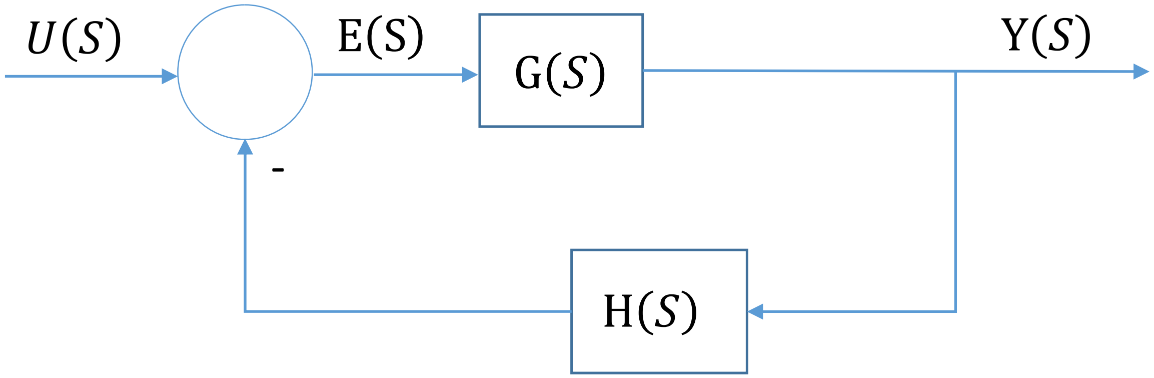
Root Locus Method

Example 1: A simplified form of the open-loop transfer function of an airplane with an autopilot in the longitudinal mode is:
a) Sketch the root-locus plot for the system.

$$G(S)H(S) = \frac{K(S+2)}{S^2+2S+3}$$



$$\frac{Y(S)}{U(S)} = \frac{G(S)}{1+G(S)H(S)}$$ Transfer function

$$P(S) = 1 + G(S)H(S) = 0$$ Characteristic equation

$$G(S)H(S) = -1$$

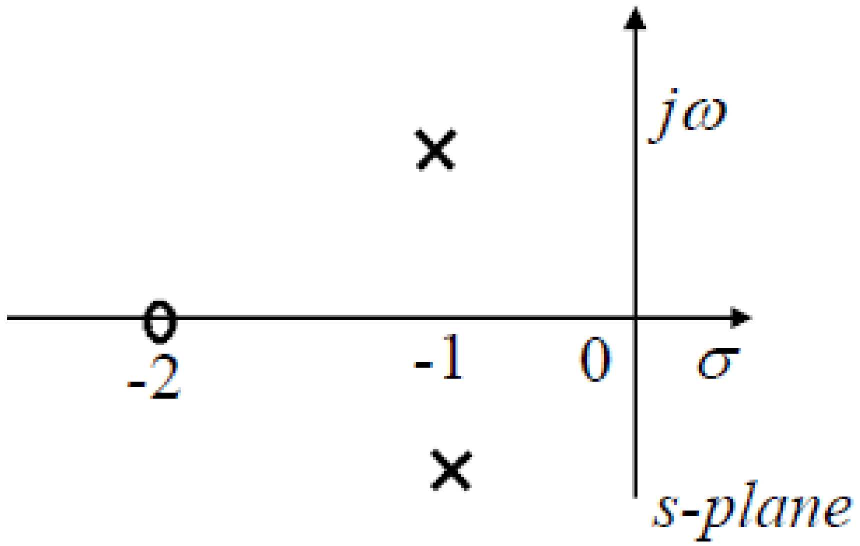
$$G(S)H(S) = \frac{K(S+2)}{S^2+2S+3}$$ Open Loop Transfer function

Root Locus Method

1. Locate the poles and zeros of $G(s)H(s)$ on the s plane: The first step in constructing a root-locus plot is to locate the open-loop poles $G(s)H(s)$;

$S_{1,2} = -1 \pm j\sqrt{2}$

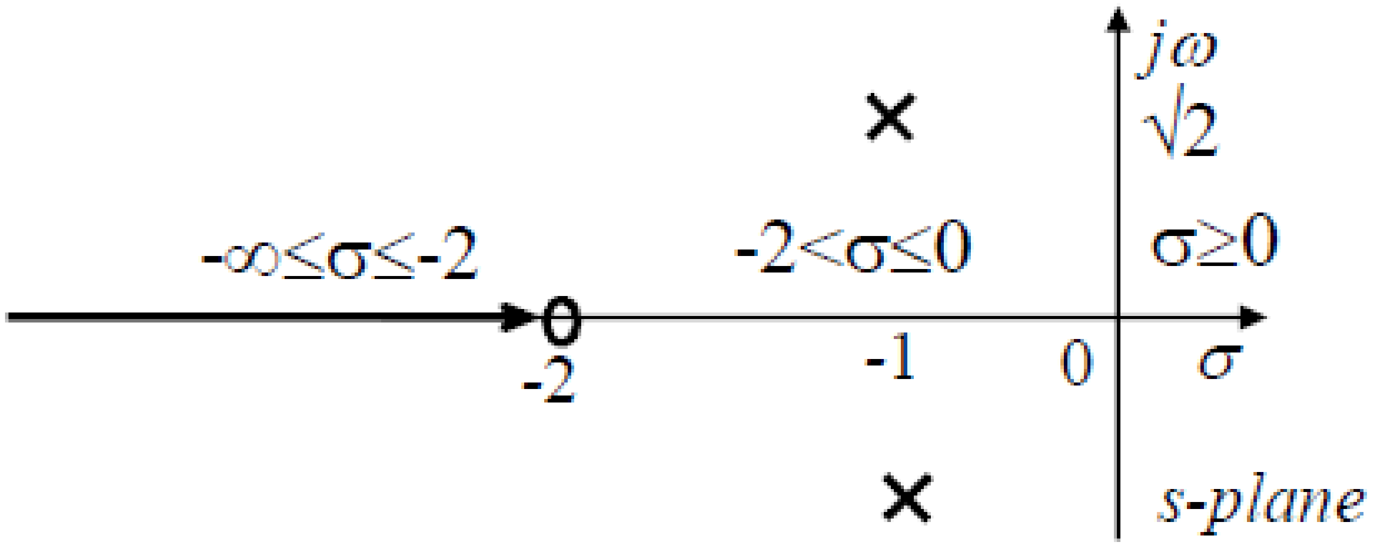
The open-loop poles $G(s)H(s)$; $Z_1 = -2$



Root Locus Method

2. Determine the root loci on the real axis: Root loci exist on the real axis between -2 and -∞.

$$G(S)H(S) = \frac{K(S+2)}{S^2+2S+3}$$



Root Locus Method

3. Determine the asymptotes of root loci: Since n=2 and m=1, Since there are two open-loop poles and one zero, there is one asymptote, which coincides with the negative real axis.

4. Find the breakaway and break-in points : Depending on the characteristic equation P(s).

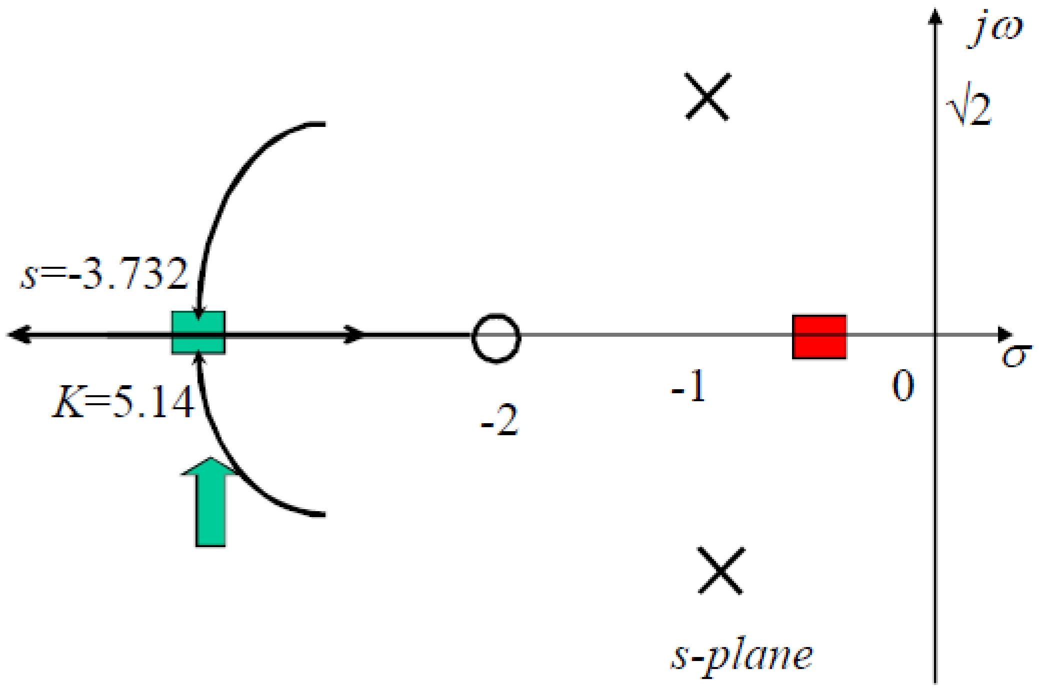
The characteristic equation is given by : $P(S) = 1 + K\frac{(S+2)}{S^2+2S+3} = 0 \qquad K = -\frac{S^2+2S+3}{(S+2)}$

By differentiating K with respect to S, we get : $\frac{dK}{dS} = -\frac{2(S+1)(S+2)-(S^2+2S+3)}{(S+2)^2} = 0 \Rightarrow \begin{cases} S_1 = -0.268 \\ S_2 = -3.732 \end{cases}$

The point $S_2 = -3.7320$ is on the root locus. Hence this point is an actual break-in point. (Note that at point $S_2 = -3.7320$ the corresponding gain value is $K = 5.14$.) Since point $S_1 = -0.2680$ is not on the root locus, it cannot be a break-in point. (For point $S_1 = -0.2680$, the corresponding gain value is $K = -1.4641$.)

Root Locus Method

$S_2 = -3.7320$ is a break-in point.



Root Locus Method

5. Determine the angle of departure (angle of arrival) of the root locus from a complex pole (at a complex zero).

For the open-loop poles $G(s)H(s)$ at $S_{3,4} = -1 \pm j\sqrt{2}$, the angle of departure θ is

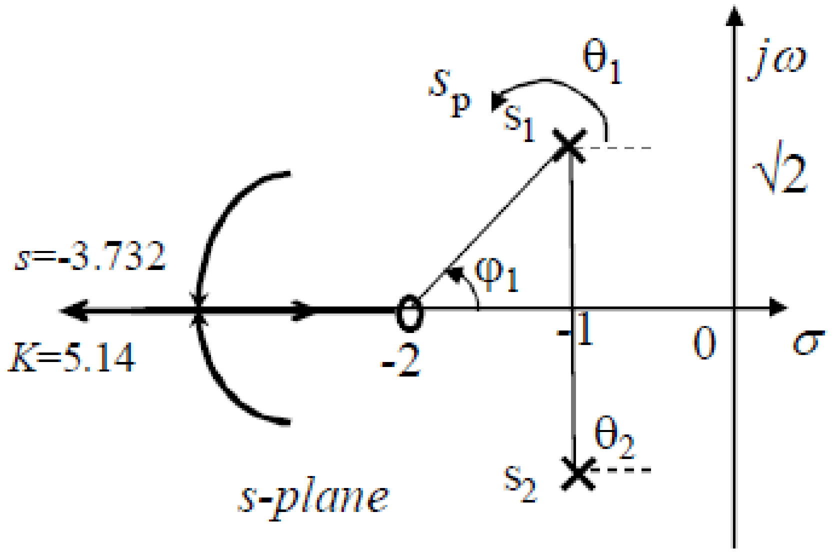
Angle of departure from a complex pole

$$\theta = 180^{\circ} - \sum_{j=1}^{n-1} \theta_j + \sum_{i=1}^{m} \varphi_i$$

$$\theta = 180^{\circ} - 90^{\circ} + \tan^{-1}\sqrt{2}$$

$$\theta = 180^{\circ} - 90^{\circ} + 54.73^{\circ}$$

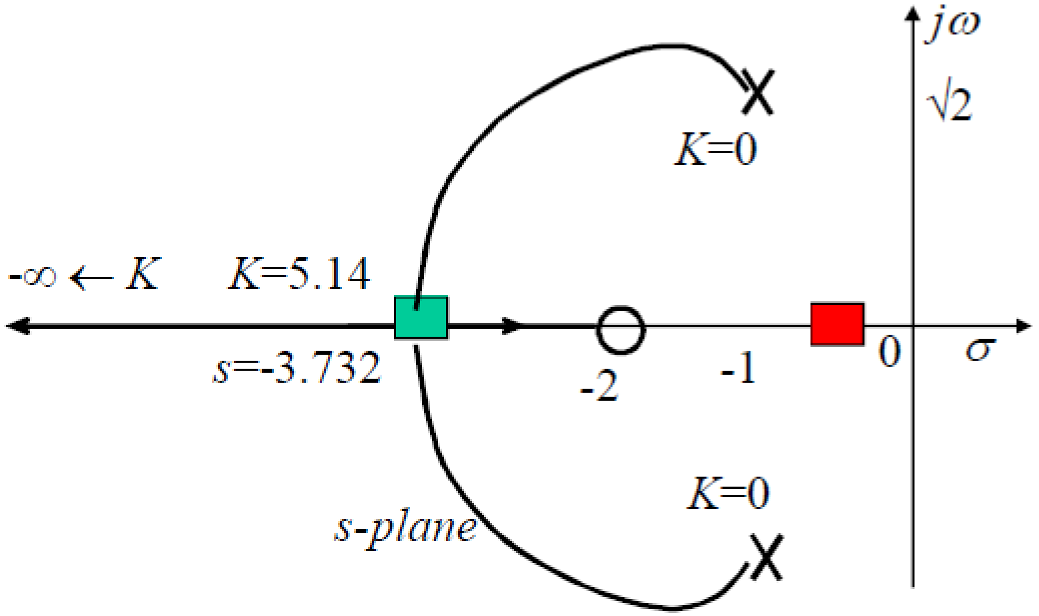
$$\theta = 144.73^{\circ}$$



Root Locus Method

6. Find the points where the root loci may cross the imaginary axis: Since the system has $n - m < 3$ then there is no loci to cross the imaginary axis.

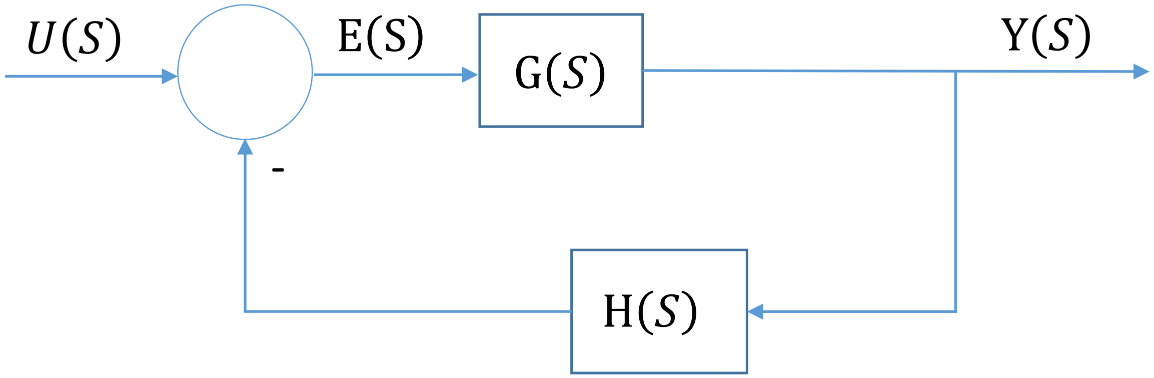
In this system the root locus in the complex plane is a part of a circle. Such a circular root locus will not occur in most systems. Circular root loci may occur in systems that involve two poles and one zero, two poles and two zeros, or one pole and two zeros. Even in such systems, whether circular root loci occur depends on the locations of poles and zeros involved.



Root Locus Method

HOMEWORK

Homework 1: Consider the system shown in Figure where system blocks transfer function are



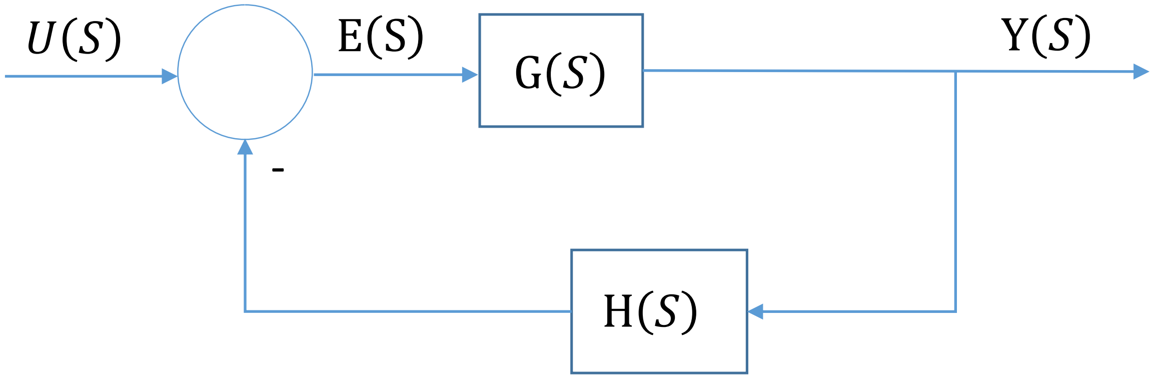
$$G(S) = \frac{K(S+2)}{S^2(S+10)} \qquad H(S) = 1$$

Plot the root loci for the closed-loop control system?

Root Locus Method

HOMEWORK

Homework 2: Consider the system shown in Figure where system blocks transfer function are



$$G(S) = \frac{K(S+9)}{S^2(S^2+4S+10)} \qquad H(S) = 1$$

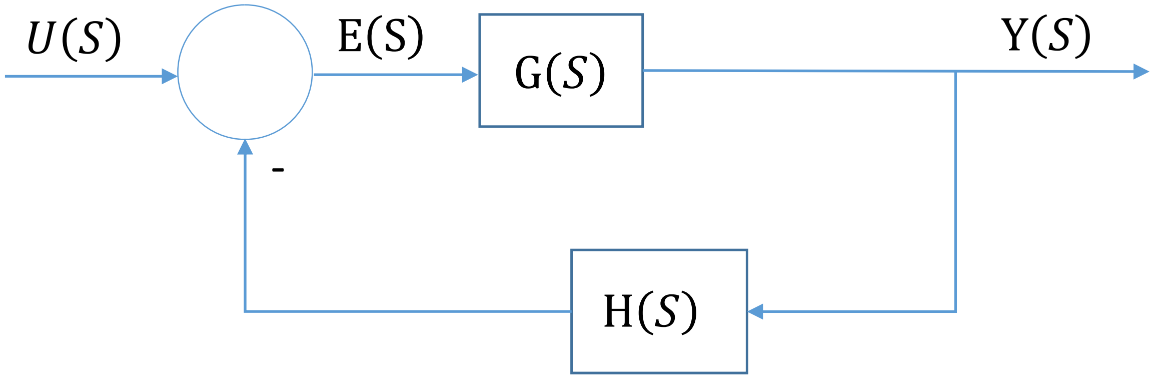
a) Plot the root loci for the closed-loop control system?
b) Determine the range of gain K for stability?

Root Locus Method

HOMEWORK

Homework 3: Consider the system shown in Figure where system blocks transfer function are

$$G(S) = \frac{10}{S(S+1)} \qquad H(S) = (1 + KS)$$

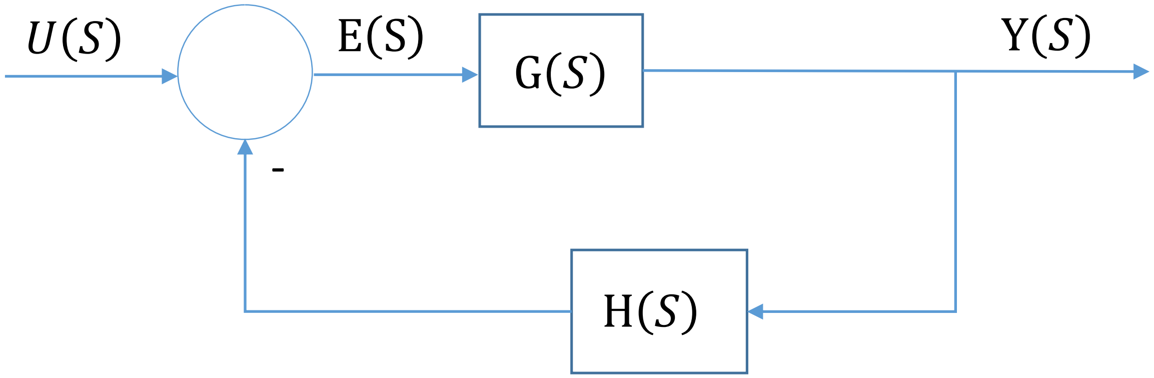


Plot the root loci for the closed-loop control system?

Root Locus Method

HOMEWORK

Homework 4: Consider the system shown in Figure where system blocks transfer function are



$$G(S) = \frac{K(S+1)}{S^2(S+2)(S+5)} \qquad H(S) = 1$$

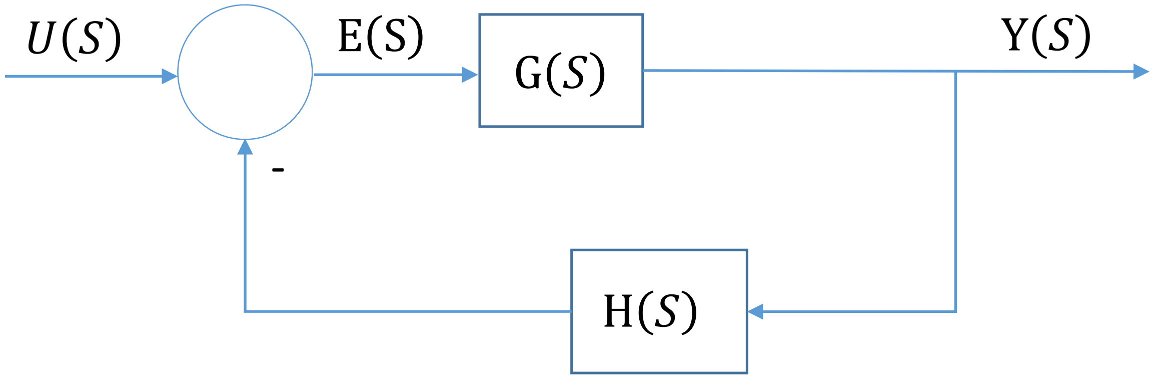
a) Plot the root loci for the closed-loop control system?
b) Determine the range of gain K for stability?

Root Locus Method

HOMEWORK

Homework 5: Consider the system shown in Figure where system blocks transfer function are

$$G(S) = \frac{K(S+1)}{S^2(S^2+2S+6)} \qquad H(S) = \frac{1}{S+1}$$



a) Plot the root loci for the closed-loop control system?
b) Determine the range of gain K for stability?