Bilad Alrafidain University College

Electric Power Techniques Engineering Department

Control Systems Analysis

Fourth Stage

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Lecture Ten

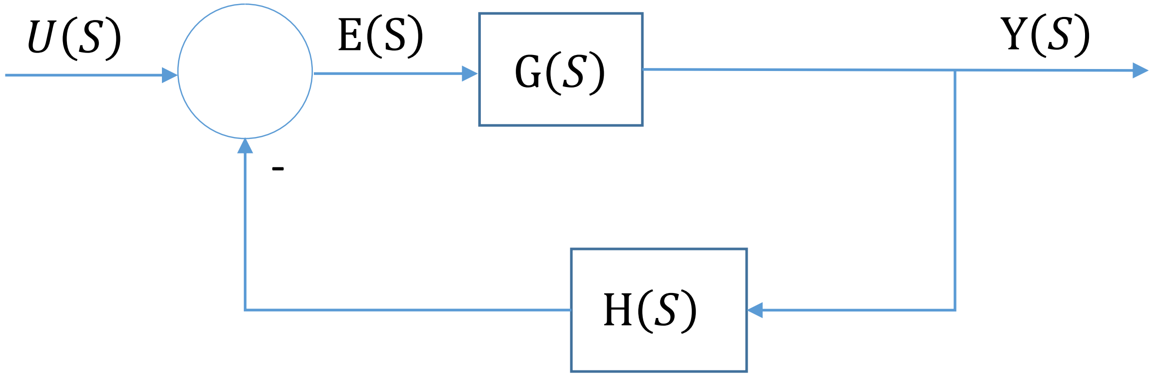
Root Locus Method

Assistant Lecturer. Ibrahim Ismail

Root Locus Method

$$\frac{Y(S)}{U(S)} = \frac{G(S)}{1 + G(S)H(S)}$$ Transfer function

$$P(S) = 1 + G(S)H(S)$$ Characteristic equation



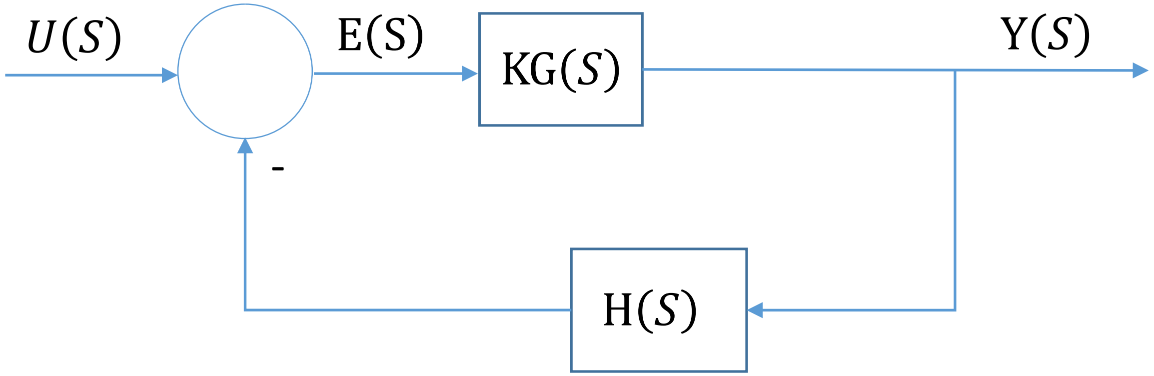
The closed-loop transfer function poles are the roots of the characteristic equation.

The basic characteristic of the transient response of a closed-loop system is closely related to the location of the closed-loop transfer function poles.

Root Locus Method

$$\frac{Y(S)}{U(S)} = \frac{KG(S)}{1 + KG(S)H(S)}$$ Transfer function

$$P(S) = 1 + KG(S)H(S) = 0$$ Characteristic equation

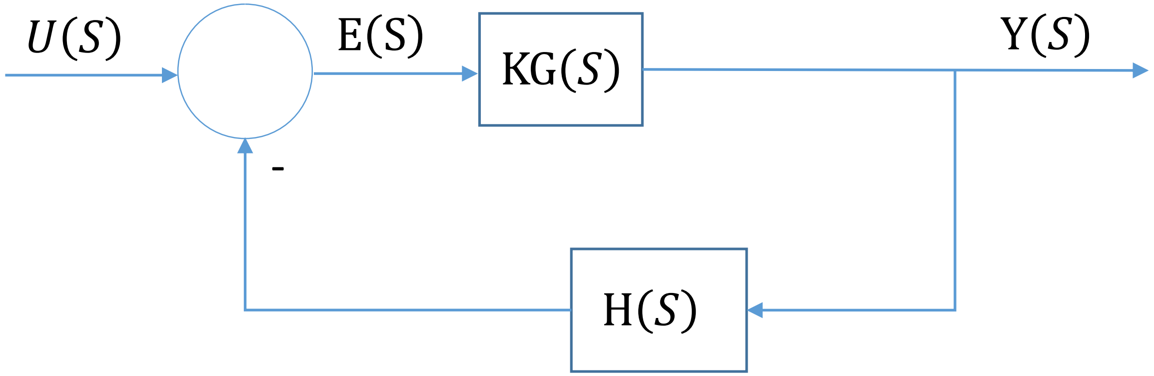


If the system has a variable loop gain, then the location of the closed-loop poles depends on the value of the loop gain chosen. It is important, therefore, that the designer know how the closed-loop poles move in the *s* plane as the loop gain is varied.

Root Locus Method

$$\frac{Y(S)}{U(S)} = \frac{KG(S)}{1 + KG(S)H(S)}$$ Transfer function

$$P(S) = 1 + KG(S)H(S) = 0$$ Characteristic equation



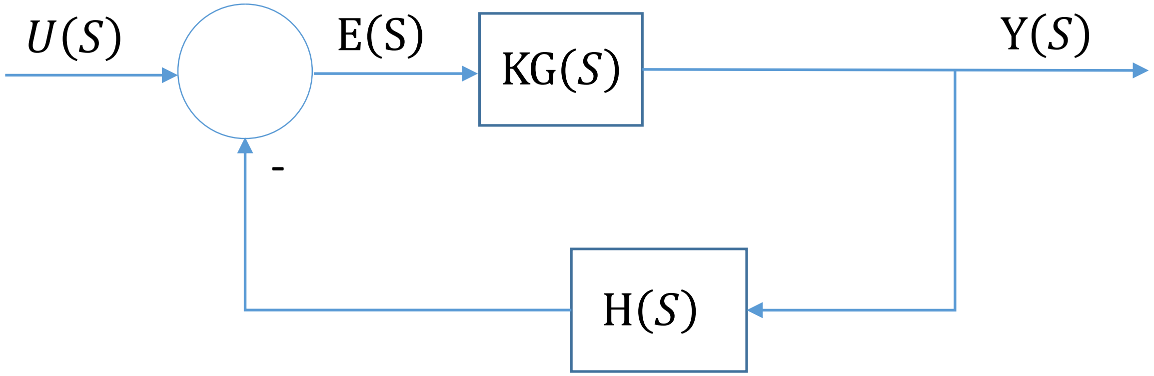
Finding the roots of the characteristic equation of degree higher than 3 is laborious and will need computer solution. MATLAB provides a simple solution to this problem.

Finding the roots of the characteristic equation may be of limited value, because as the gain of the open-loop transfer function varies the characteristic equation changes and the computations must be repeated.

Root Locus Method

$$\frac{Y(S)}{U(S)} = \frac{KG(S)}{1 + KG(S)H(S)}$$ Transfer function

$$P(S) = 1 + KG(S)H(S) = 0$$ Characteristic equation



A simple method for finding the roots of the characteristic equation has been developed by Prof. W. R. Evans in 1948 and used extensively in control engineering. This method, called the *root-locus method*, is one in which the roots of the characteristic equation are plotted in complex $S = \sigma \pm j\omega$ plane for all values of a system parameter.

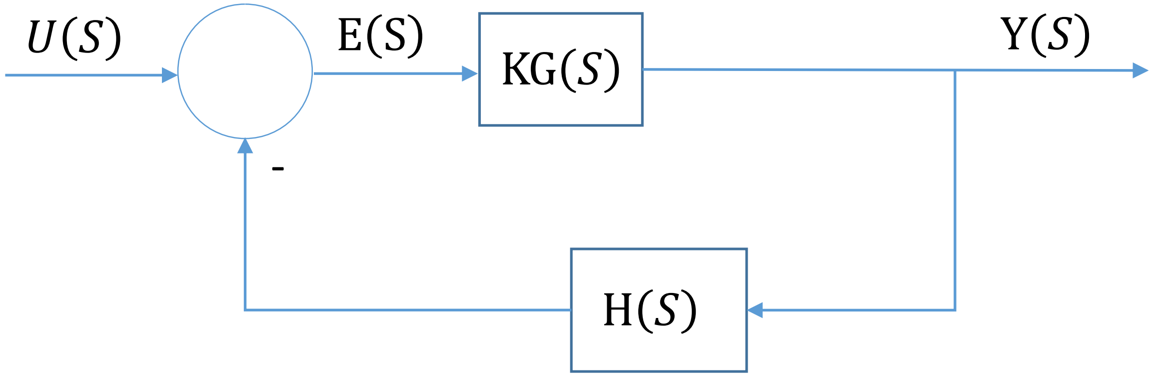
The parameter is usually the gain, but any other variable of the open-loop transfer function may be used.

Unless otherwise stated, we shall assume that the gain of the open-loop transfer function is the parameter to be varied through all values, from zero to infinity.

Root Locus Method

$$\frac{Y(S)}{U(S)} = \frac{KG(S)}{1+KG(S)H(S)}$$ Transfer function

$$P(S) = 1 + KG(S)H(S) = 0$$ Characteristic equation



The root locus is the locus of roots of the characteristic equation of the closed loop system as a specific parameter (usually, gain K) is varied from zero to infinity, $0 \leq K \leq \infty$ giving the method its name.

The root locus plot shows the contributions of each open-loop pole or zero $G(s)H(s)$ to the locations of the closed-loop poles $P(s) = 1 + G(s)H(s) = 0$.

Root Locus Method

In designing a linear control system, we find that the root-locus method proves quite useful since it indicates the manner in which the open-loop poles and zeros should be modified so that the response meets system performance specifications.

This method is particularly suited to obtaining approximate results very quickly. By using the root-locus method, it is possible to determine the value of the loop gain K that will make the damping ratio of the dominant closed-loop poles as prescribed.

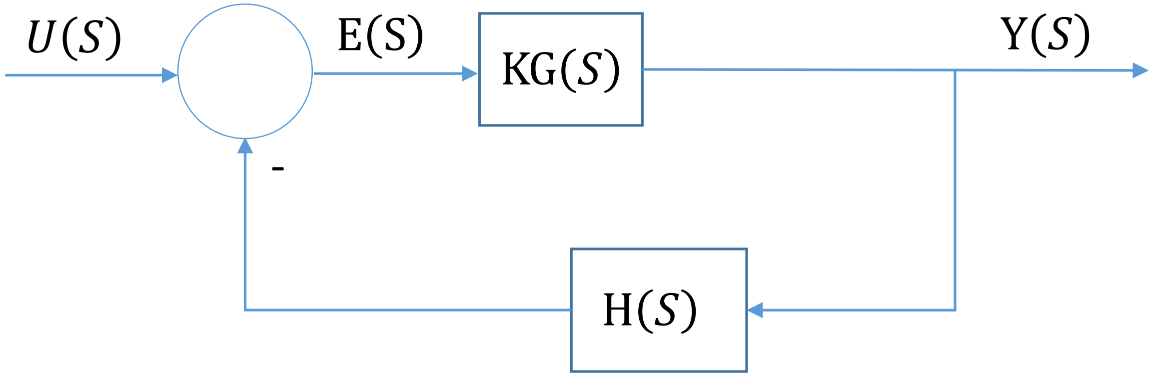
The root locus is the path of the roots of the characteristic equation traced out in the s-plane as a system parameter varies from zero to infinity.

Root Locus Method

$$\frac{Y(S)}{U(S)} = \frac{KG(S)}{1 + KG(S)H(S)}$$

$$P(S) = 1 + KG(S)H(S) = 0$$

$$KG(S)H(S) = -1$$



Angle Condition: $\angle KG(S)H(S) = \pm 180^{\circ}(2K + 1), (K = 0,1,2, \ldots)$

Magnitude Condition: $|KG(S)H(S)| = 1$

The characteristic equation can be split into two equations by equating the **angles** and **magnitudes** of both sides, respectively, to obtain the following:

Root Locus Method

Angle Condition: $\angle KG(S)H(S) = \pm 180^{\circ}\,(2K+1), (K = 0,1,2,\dots)$

Magnitude Condition: $|KG(S)H(S)| = 1$

The values of *s* that fulfil both the **angle** and **magnitude** conditions are the roots of the characteristic equation, or the closed-loop poles.

A locus of the points in the complex plane satisfying the **angle condition alone** is the root locus.

The roots of the characteristic equation (the closed-loop poles) corresponding to a given value of the gain can be determined from the magnitude condition.