Bilad Alrafidain University College

Electric Power Techniques Engineering Department

Control Systems Analysis

Fourth Stage

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Lecture Nine

Stability Analysis

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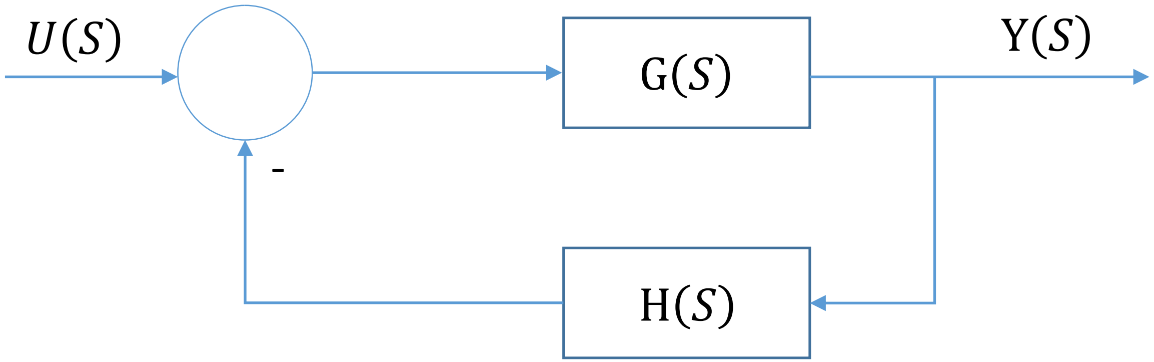
Routh-Hurwitz stability criterion: Special Cases

$$\frac{Y(S)}{U(S)} = \frac{G(S)}{1 + G(S)H(S)}$$

$$P(S) = 1 + G(S)H(S) = 0$$



Case One: If a first-column term in any row is zero, but the remaining terms are not zero or there is no remaining term, then the zero term is replaced by a very small positive number $\varepsilon > 0$ and the rest of the array is evaluated.

Routh-Hurwitz stability criterion: Special Cases

Example 1. Consider the following characteristic equation:

$$P(S) = s^5 + 2s^4 + 2s^3 + 4s^2 + 11s + 10 = 0$$

$$b_1 = \frac{2*2 - 1*4}{2} = 0, b_1 = \varepsilon, \varepsilon > 0$$

$$c_1 = \frac{4*\varepsilon - 2*6}{\varepsilon} = 4 - \frac{12}{\varepsilon} \approx \frac{12}{\varepsilon} < 0$$

$$d_1 = \frac{6*c_1 - 10*\varepsilon}{c_1} = 6 - \frac{10*\varepsilon}{c_1} \approx 6 > 0$$

Routh-Hurwitz array:

s^5	1	2	11
s^4	2	4	10
s^3	b_1	6	0
s^2	c_1	10	0
s^1	d_1	0	0
s^0	10	0	0

Routh-Hurwitz stability criterion: Special Cases

$P(S) = s^5 + 2s^4 + 2s^3 + 4s^2 + 11s + 10 = 0$

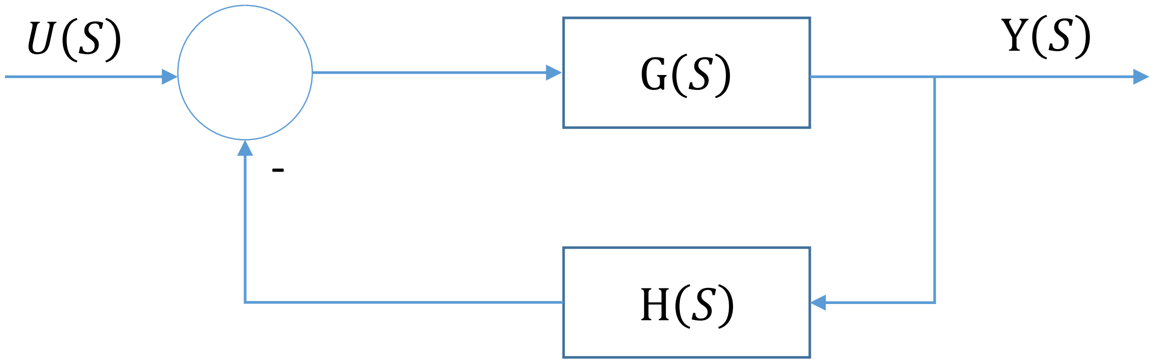
s^5	1	2	11
s^4	2	4	10
s^3	ε	6	0
s^2	$c_1 = 4 - \frac{12}{\varepsilon} \approx \frac{12}{\varepsilon} < 0$	10	0
s^1	$d_1 = 6 - \frac{10*\varepsilon}{c_1} \approx 6 > 0$	0	0
s^0	10	0	0

There are two sign changes in the first column due to the large negative number calculated for $c1$. Thus, the system is unstable because two roots lie in the right half of the plane.

Routh-Hurwitz stability criterion: Special Cases

$$\frac{Y(S)}{U(S)} = \frac{G(S)}{1 + G(S)H(S)}$$

$$P(S) = 1 + G(S)H(S) = 0$$



Case Two: If all the coefficients in any derived row are zero, it indicates:
There are roots of equal magnitude lying radially opposite in the *s-plane*, that is, two real roots with equal magnitudes and opposite signs and/or two conjugate imaginary roots. Hence, the system is **unstable.**

However, the evaluation of the rest of the array can be continued to obtain the pole/poles on the right hand side of *s-plane* by forming an auxiliary polynomial with the coefficients of the last row and by using the coefficients of the derivative of this polynomial in the next row.

Routh-Hurwitz stability criterion: Special Cases

Example 2. Consider the following characteristic equation:

$$P(S) = s^3 + s^2 + 4s + 4 = 0$$

The terms in the s^1 row are all zero. (Note that such a case occurs only in an odd numbered row.) The auxiliary polynomial is then formed from the coefficients of the s^2 row. The auxiliary polynomial $P(s)$ is

$$P_y(s) = s^2 + 4$$

Solving $P_y(s)$ leads $s^1 = j2$ & $s^2 = -j2$ two complex conjugate roots on the imaginary axis. Thus, the system is **unstable**.

The evaluation of the rest of the array results will be : $\dfrac{dP_y(s)}{ds} = 2s$

Routh-Hurwitz array:

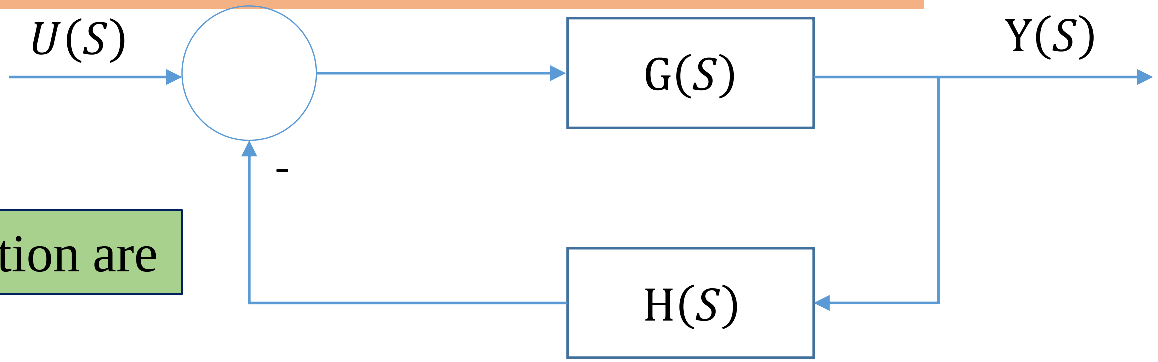
s^3	1	4
s^2	1	4
s^1	0	0
s^0		

s^1	2	0
s^0	4	

Stability of Linear Feedback Systems by Routh-Hurwitz Stability Criterion

Example 3. Determine its stability with Routh-Hurwitz stability criterion for the following system.

$$G(S) = \frac{1}{s^3 + s^2 + 2s + 23} \qquad H(S) = 1$$



The closed-loop transfer function and characteristic equation are

$$\frac{Y(S)}{U(S)} = \frac{G(S)}{1 + G(S)H(S)} = \frac{1}{s^3 + s^2 + 2s + 24}$$

$$P(S) = s^3 + s^2 + 2s + 24 = 0$$

Routh-Hurwitz array:

s^3	1	2
s^2	1	24
s^1	-22	0
s^0	24	

1’st sign change (between s^2 and s^1)

2’nd sign change (between s^1 and s^0)

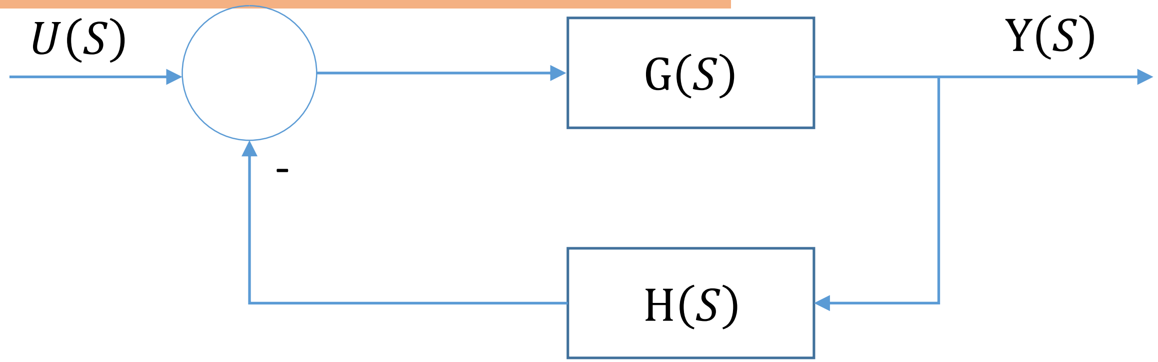
The system has two poles on the right hand side of *s-plane*.
Hence it is **unstable.**

Stability of Linear Feedback Systems by Routh-Hurwitz Stability Criterion

Routh-Hurwitz stability criterion can be used to determine the effects of changing one or two parameters of a system by examining the values that cause instability.

Stability of Linear Feedback Systems by Routh-Hurwitz Stability Criterion

Example 3. Consider the system shown in Figure. Determine the range of *K* for stability.



$$\frac{Y(S)}{U(S)} = \frac{G(S)}{1+G(S)H(S)} = \frac{K}{s(s^2+s+1)(s+2)K}$$

$$P(S) = s^4 + 3s^3 + 3s^2 + 2s + K = 0$$

Routh-Hurwitz array:

s^4	1	3	K
s^3	3	2	0
s^2	7/3	K	
s^1	2-9K/7		
s^0	K		

For stability, *K* must be positive, and all coefficients in the first column must be positive. Therefore,

$$0 < K < \frac{14}{9}$$

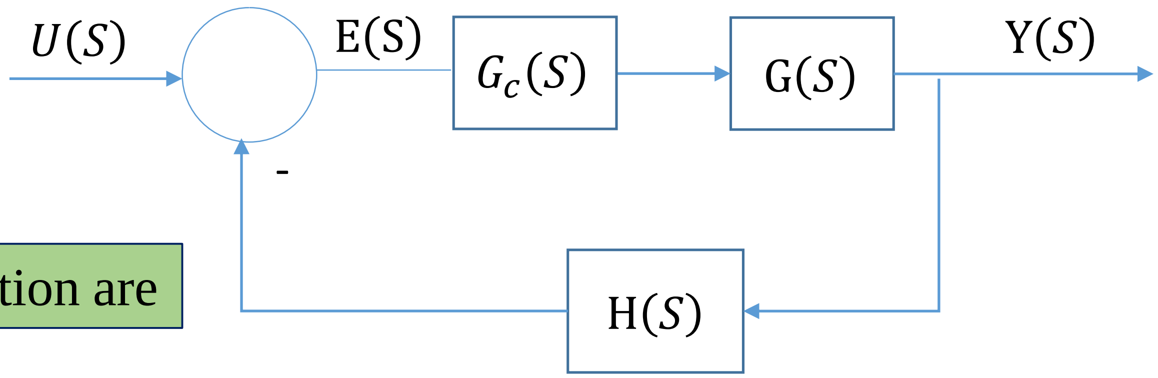
The system is stable for $0 < K < \frac{14}{9}$

Stability of Linear Feedback Systems by Routh-Hurwitz Stability Criterion

Example 4. Consider the system shown in Figure. Determine the range of K for stability.

$G_c(S) = K$ $\qquad H(S) = 1$

$$G(S) = \frac{1}{s^3 + s^2 + 4s}$$



The closed-loop transfer function and characteristic equation are

$$\frac{Y(S)}{U(S)} = \frac{G(S)}{1 + G(S)H(S)} = \frac{K}{s^3 + 2s^2 + 4s + K}$$

$$P(S) = s^3 + 2s^2 + 4s + K = 0$$

Routh-Hurwitz array:

s^3	1	4
s^2	2	K
s^1	8-K/2	0
s^0	K	

$0 < K < 8$

For stability, K must be positive, And all coefficients in the first column must be positive. Therefore,

The system is stable for $0 < K < 8$