Bilad Alrafidain University College

Electric Power Techniques Engineering Department

Control Systems Analysis

Fourth Stage

Academic Year 2020 - 2021

Lecture Seven

Stability Analysis

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Stability of Control Systems

Stability of closed-loop feedback systems is central to control system design.

A control system is in equilibrium if:

- The absence of any disturbance or input.
- The output stays in the same state.

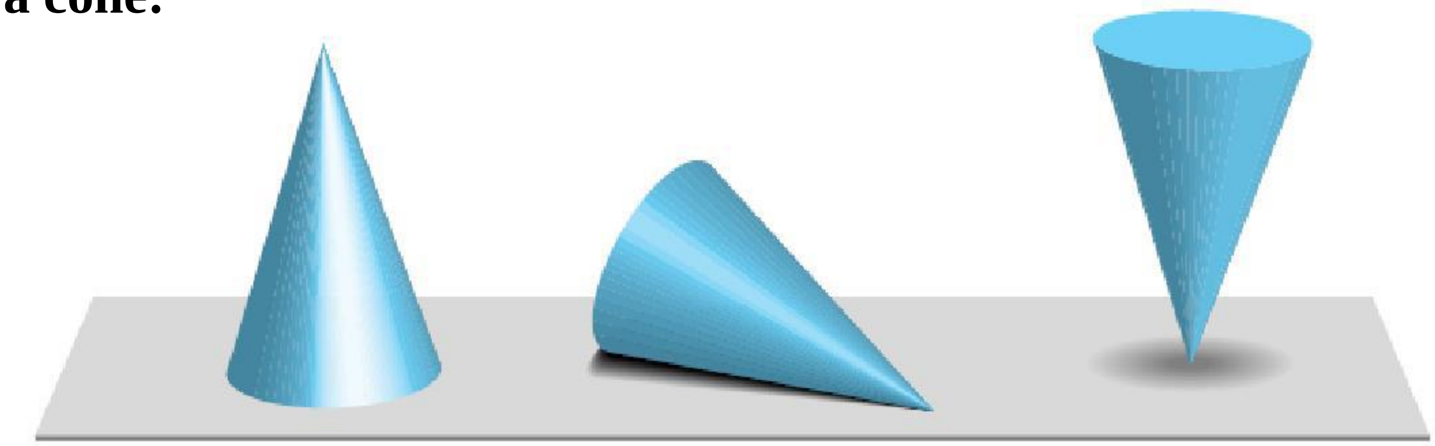
يعد إستقرار أنظمة التغذية العكسية ذات المسار المغلق أمراً أساسياً للتحكم في تصميم النظام.

يكون نظام التحكم في حالة توازن إذا:

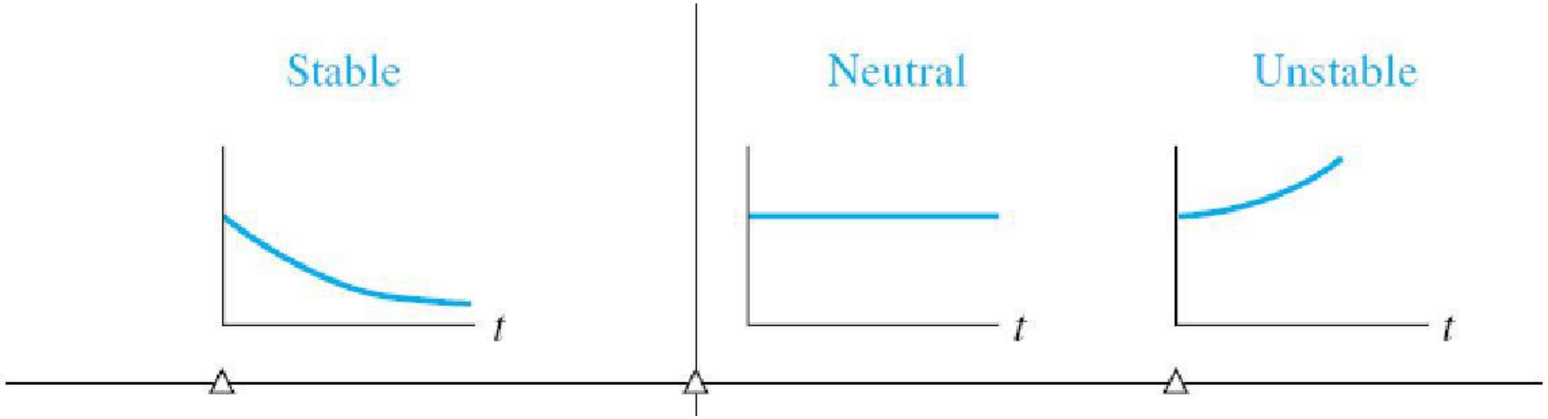
- عدم وجود أي إزعاج أو إدخال للنظام.
- الإخراج من النظام يبقى في نفس الحالة.

Stability of Control Systems

The stability of a cone:



(a) Stable (b) Neutral (c) Unstable



Stability of Control Systems

The original Tacoma Narrows Bridge was built in 1939 in the Washington State, U.S.A. At the time, it was the third longest suspension bridge in the world, and cost about US$6.56 million (considered a bargain then). The Tacoma Narrows Bridge opened to the public on July 1, 1940.

Stability of Control Systems

Opened on July 1, 1940. On November 7, 1940, a windstorm induced severe torsional movement in the bridge that eventually caused the bridge to break apart and collapse.

Stability of Control Systems

1. Stable

A linear time-invariant control system is stable if the output eventually comes back to its equilibrium state when the system is subjected to an initial condition.

يكون نظام التحكم الخطي غير المتغيرمع الزمن مستقراً إذا عاد الإخراج في النهاية إلى حالة توازنه عندما يخضع النظام لشرط أولي.

Stability of Control Systems

2. Critically Stable

A linear time-invariant control system is critically stable if oscillations of the output continue forever.

يكون نظام التحكم الخطي غير المتغيرمع الزمن مستقرًا بشكل حرج إذا استمرت تذبذبات الإخراج إلى الأبد.

Stability of Control Systems

3. Unstable

A linear time-invariant control system is unstable if the output diverges without bound from its equilibrium state when the system is subjected to an initial condition.

يكون نظام التحكم الخطي غير المتغير مع الزمن غير مستقراً إذا تباعد الإخراج دون تقييد من حالة توازنه عندما يخضع النظام لشرط مبدئي.

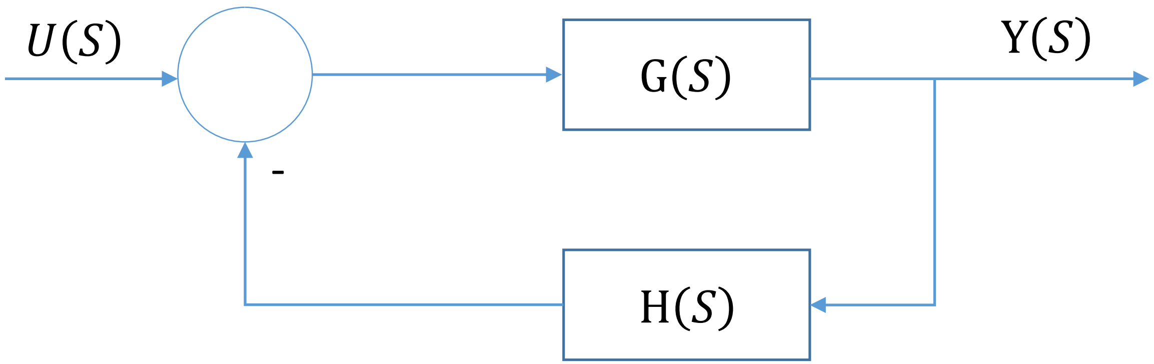
Stability of Control Systems

Note:
Actually, the output of a physical system may increase to a certain extent but may be limited by mechanical "stops," or the system may break down or become nonlinear after the output exceeds a certain magnitude so that the linear differential equations no longer apply.

ملاحظة:
في الواقع ، قد يزداد إخراج النظام المادي إلى حد معين ولكن قد يكون محدوداً بواسطة "نقاط توقف ميكانيكية"، أو قد ينهار النظام أو يصبح غير خطي بعد أن يتجاوز إخراج النظام قيمة معينة بحيث لا يتم تطبيق المعادلات التفاضلية الخطية.

Stability Methods for Control Systems

$$\frac{Y(S)}{U(S)} = \frac{G(S)}{1 + G(S)H(S)}$$

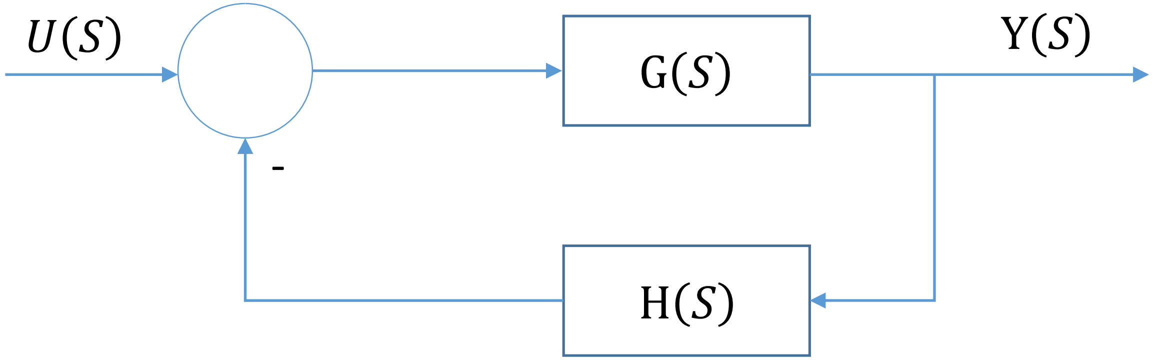


- Solving *characteristic equation*, $P(s) = 0$ or matrix $P(s) = |sI - A| = 0$, to determine the stability based on the location of roots on the complex $S = \sigma \pm j\omega$ plane.
- Applying *Routh-Hurwitz stability criterion* (*Routh* 1872, *Hurwitz* 1892), without solving $P(s) = 0$.
- Applying *Root Locus* (*Evans* 1948) Frequency analysis methods.
- *Bode diagram* (*Bode* 1927), $P(j\omega) = 0$.
- *Nyquist stability criterion* (1932) , without solving $P(j\omega) = 0$.
- *Lyapunov stability methods*, (Lyapunov 1892) (become known in west 1961)

Stability of Linear Feedback Systems by Solving Characteristic Equation

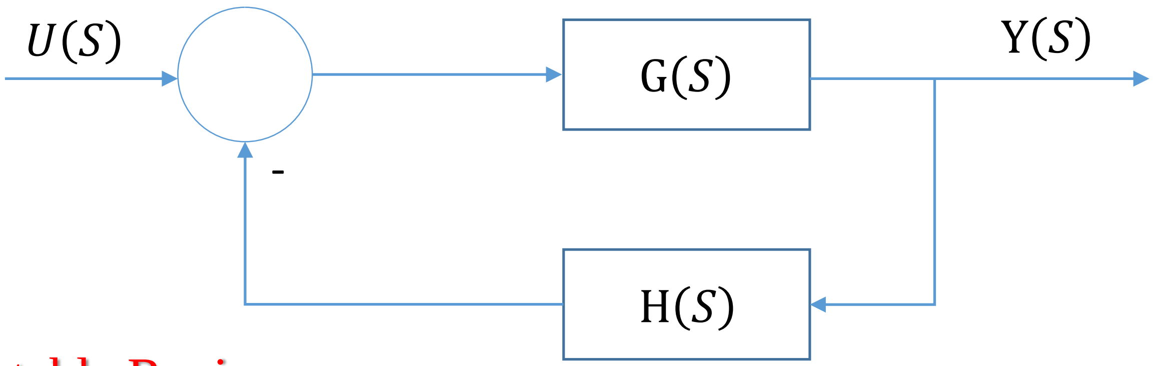
The stability of a feedback system is directly related to the location of the roots of the characteristic equation of the system transfer function and to the location of the eigenvalues of the system matrix for a system in state variable format.

Thus the stability of a linear closed-loop system can be determined from the location of the closed loop poles in the $S = \sigma \pm j\omega$ plane.

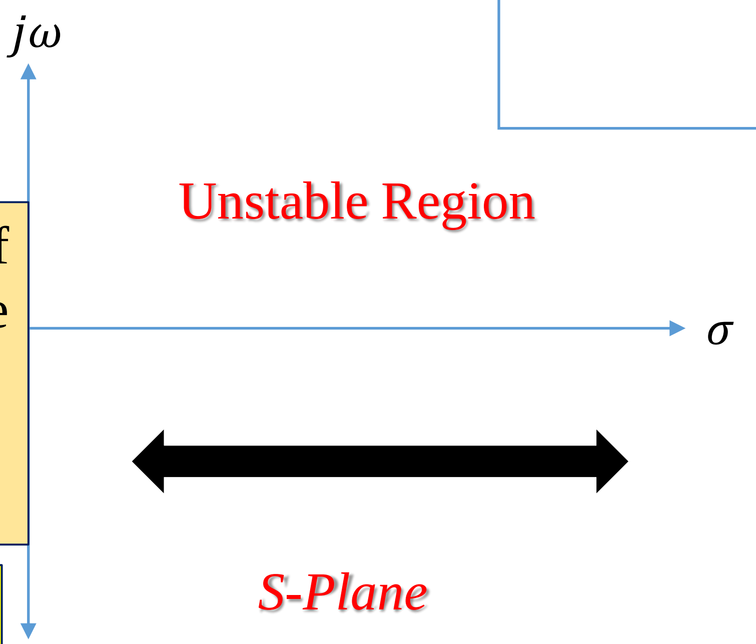


Stability of Linear Feedback Systems by Solving Characteristic Equation

$$\frac{Y(S)}{U(S)} = \frac{G(S)}{1 + G(S)H(S)}$$



$$P(S) = 1 + G(S)H(S) = 0$$



The **system is stable** if all the roots of characteristic equation $P(s)$ lie to the left of the $j\omega$ axis.

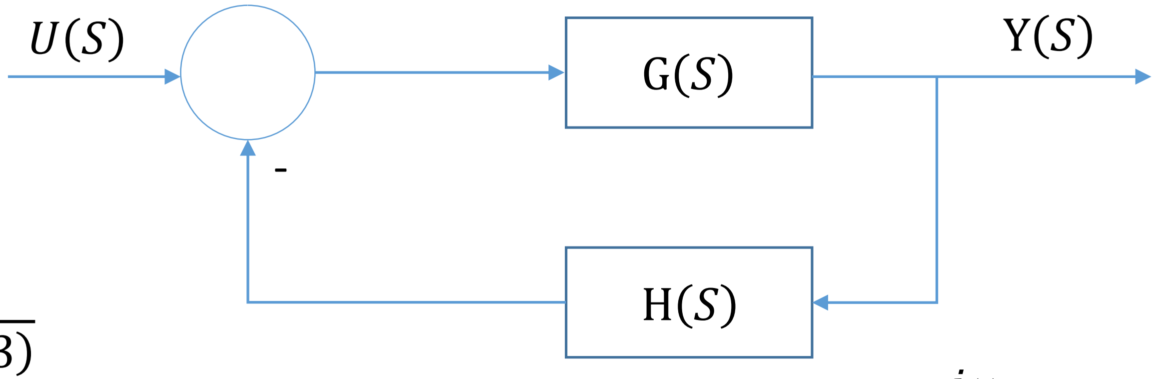
Stable region

If all roots of characteristic equation lie to the left side of the $j\omega$ axis on the complex s-plane then the system is stable. Thus the transient response of the system eventually reaches to equilibrium.

Note: Whether a linear system is stable or unstable is a property of the system itself and does not depend on the characteristic of input or driving function of the system.

Stability of Linear Feedback Systems by Solving Characteristic Equation

Example



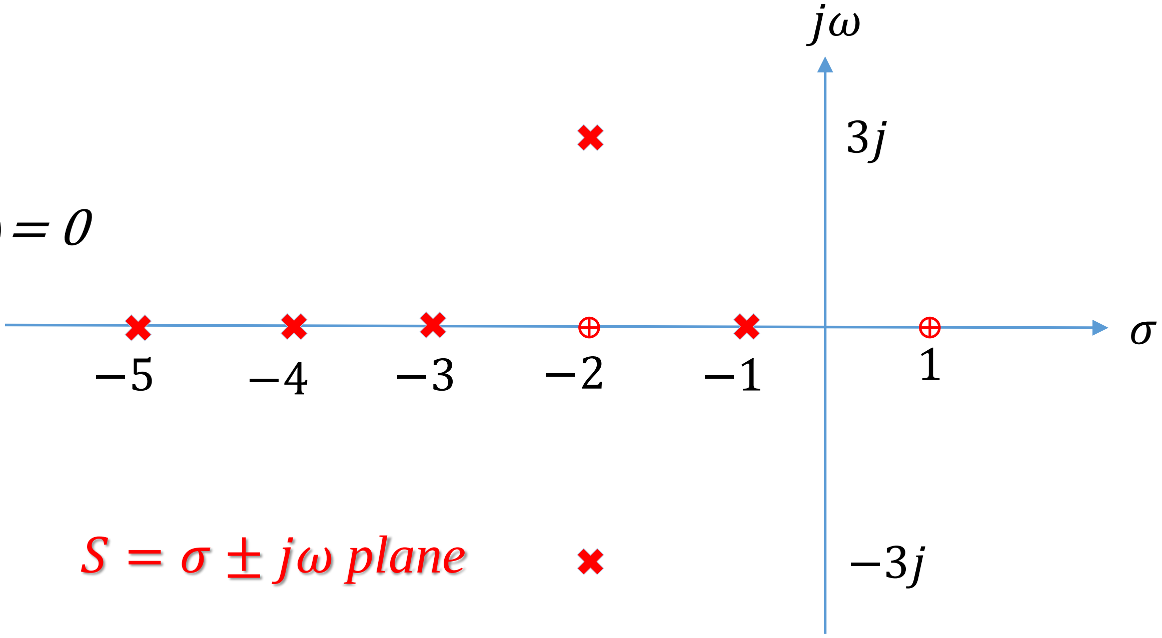
$$\frac{Y(S)}{U(S)} = \frac{G(S)}{1 + G(S)H(S)}$$

$$\frac{Y(S)}{U(S)} = \frac{(S+2)(S-1)}{(S+1)(S+4)(S+5)(s+2+j3)(s+2-j3)}$$

$$P(S) = 1 + G(S)H(S) = 0$$

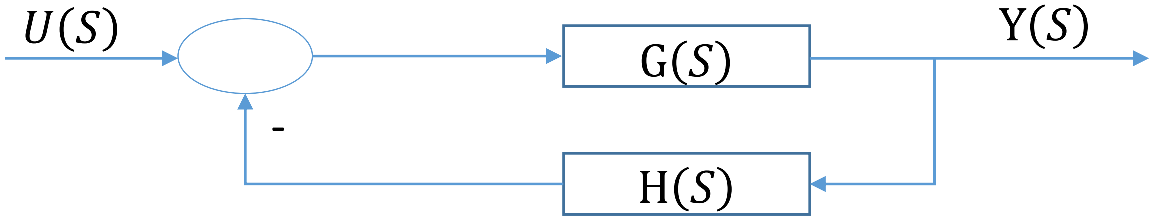
$$P(S) = (S+1)(S+4)(S+5)(s+2+j3)(s+2-j3) = 0$$

Factoring the characteristic equation or denominator polynomial of the transfer function is generally difficult and generally needs computation for $n \geq 3$.



Stability of Linear Feedback Systems by Solving Characteristic Equation

$$\frac{Y(S)}{U(S)} = \frac{G(S)}{1 + G(S)H(S)}$$



<table>
<tr><th colspan="2">State Space Model</th><th>Transfer Function Model</th></tr>
<tr><td>$\dot{x} = Ax + Bu$
$x \in R^n$
$u \in R^r$
$A \in R^{n*n}$
$B \in R^{n*r}$</td><td>$y = Cx + Du$
$C \in R^{m*n}$
$D \in R^{m*r}$</td><td rowspan="2">$\frac{Y(S)}{U(S)} = \frac{G(S)}{1 + G(S)H(S)}$
$P(s) = 1 + G(S)H(S) = 0$</td></tr>
<tr><td colspan="2">$P(s) = |sI - A| = 0$</td></tr>
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