Bilad Alrafidain University College

Electric Power Techniques Engineering Department

Control Systems Analysis

Fourth Stage

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Assistant Lecturer. Ibrahim Ismail

Control Systems Analysis

Course Contents

- Introduction to Control System.
- Transfer Function.
- Time Domain Analysis.
- Stability Analysis.
- Root Locus Method.
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- State Space Theory.
- State Space Representation.

Lecture Six

Time Domain Analysis

Relation between S-Plane and ω_n and ζ

1. Natural Undamped Frequency (ω_n)

   The distance from the origin of s-plane to the pole is the Natural Undamped Frequency ω_n in rad/sec. For example, if ω_n= 3, the pole is located anywhere on a circle with radius 3. Therefore the s-plane is divided into Constant Natural Undamped Frequency (ω_n) circles.

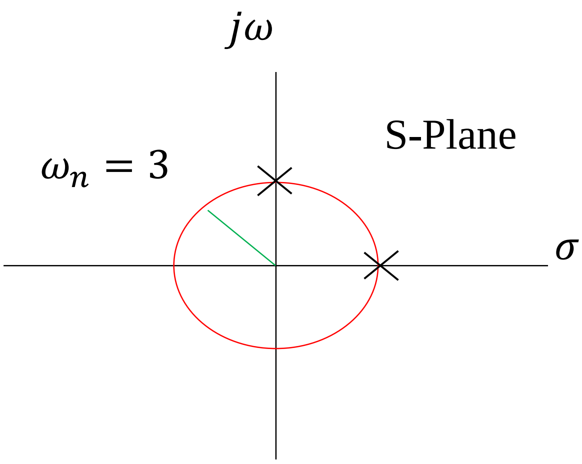


Figure 1. S-Plane when ω_n= 3

2. Damping Ratio (ζ)

   Cosine of the angle between the vector connecting origin to pole and the -ve real axis yields damping ratio. $\zeta = Cos\ (\theta)$.

   For Undamped system: $\theta = 90^{\circ}$. So that $\zeta = 0$

   For Critically damped system: $\theta = 0^{\circ}$. So that $\zeta = 1$

   The s-plane is divided into sections of constant damping ratio lines.

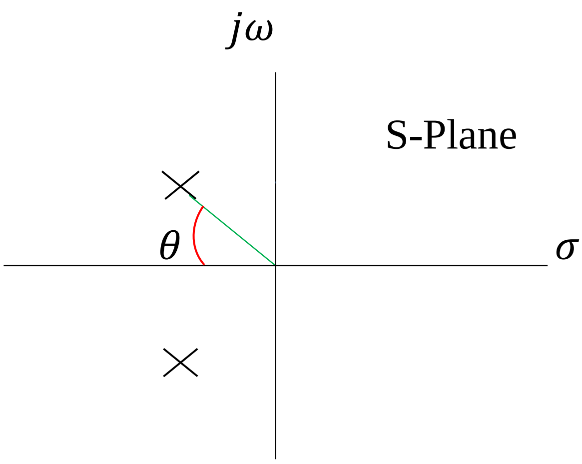


Figure 2. S-Plane showing θ

Example 1. Determine the natural frequency and damping ratio of the pole from the following PZ-map.

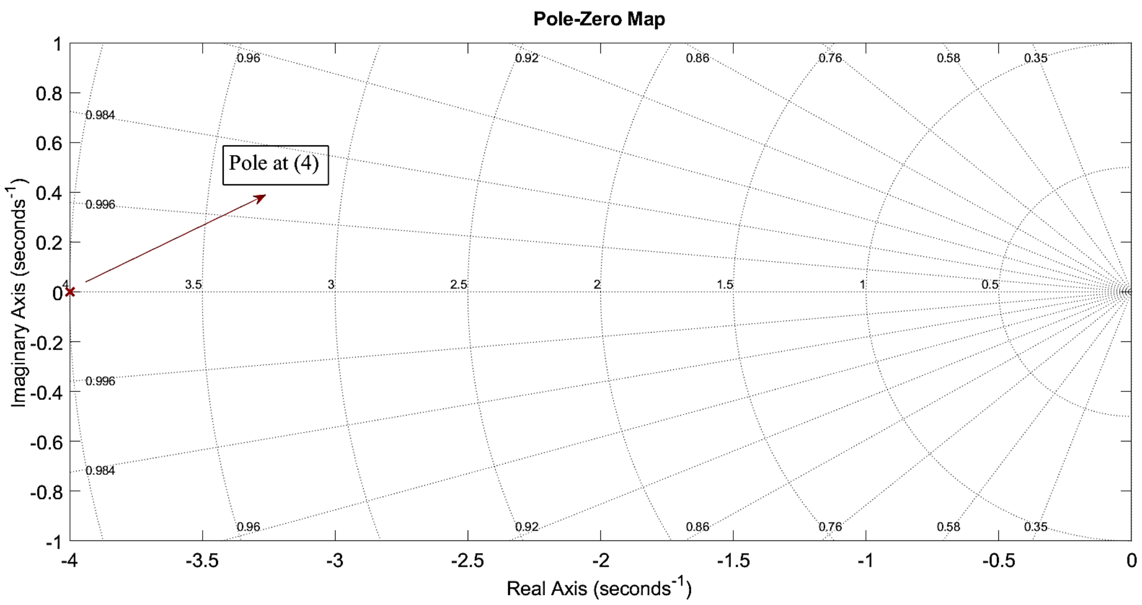


Figure 3. PZ Map

Solution

From the previous PZ map, we noticed pole at $(s = 4)$. Therefore the natural frequency and damping ratio will be equals to:

The distance from the origin of s-plane to the pole is the Natural Undamped Frequency ω_n. Therefor $(\omega_n = 4\, rad/sec)$. Because the distance from the origin of s-plane to the pole is (4).

Since the angle between the vector connecting origin to pole and the -ve real axis is zero $(\theta = 0^{\circ})$, then the $(\zeta_1 = Cos\ (0^{\circ}) = 1)$.

Example 2. Determine the natural frequency and damping ratio of the poles from the following PZ-map.

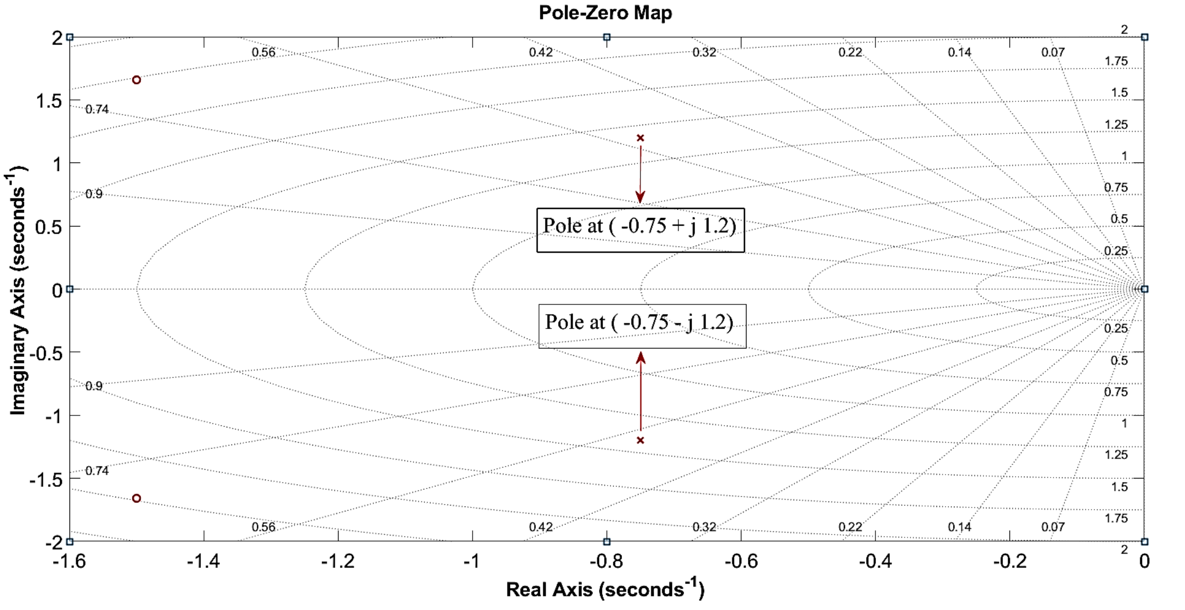


Figure 4. PZ Map

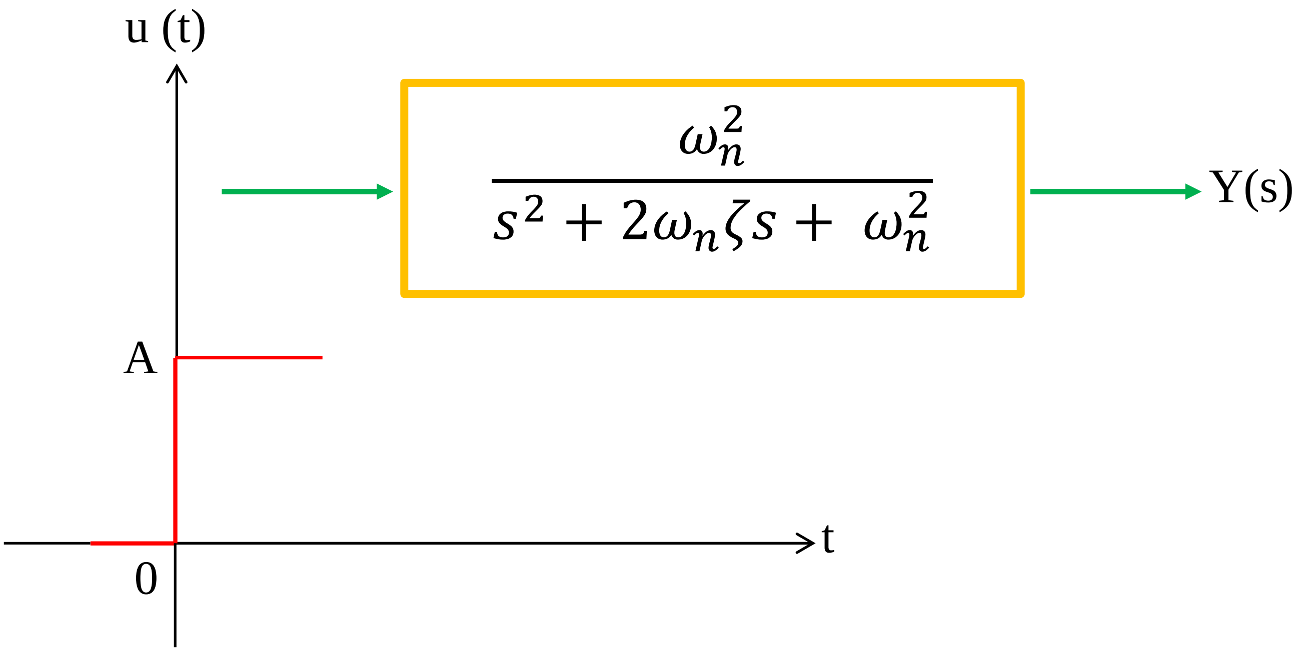
Solution

From the previous PZ map, we noticed poles at $(s_1 = -0.75 + j1.2)$ & $(s_2 = -0.75 - j1.2)$. Therefore the natural frequency and damping ratio will be equals to:

The distance from the origin of s-plane to the pole is the Natural Undamped Frequency ω_n. Therefor $(\omega_n = 1.41\, rad/sec)$. Because the distance from the origin of s-plane to the pole is (1.41).

Since the angle between the vector connecting origin to pole and the -ve real axis is $(\theta = 1.012^{\circ})$, then the $\left(\zeta = Cos\left(1.012^{\circ}\right)^{\circ} = 0.53\right)$.

Step Response of Second Order Systems



$$Y\,(s) = G\,(s)U(s) = \frac{\omega_n^2}{s(s^2 + 2\omega_n\zeta s + \omega_n^2)}$$

By partial fraction:

$$Y\,(s) = \frac{1}{s} - \frac{s + 2\zeta\omega_n}{(s + \zeta\omega_n)^2 + \omega_d^2}$$

Where, $\omega_d = \omega_n\sqrt{1 - \zeta^2}$ (Damped Natural Frequency)

Using Inverse Laplace Transform:

$$y(t) = 1 - e^{-\zeta\omega_n t}\left[\cos\omega_d t + \frac{\zeta}{\sqrt{1-\zeta^2}} sin\omega_d t\right]$$

Effect of changing ζ and ω_n on Step Response

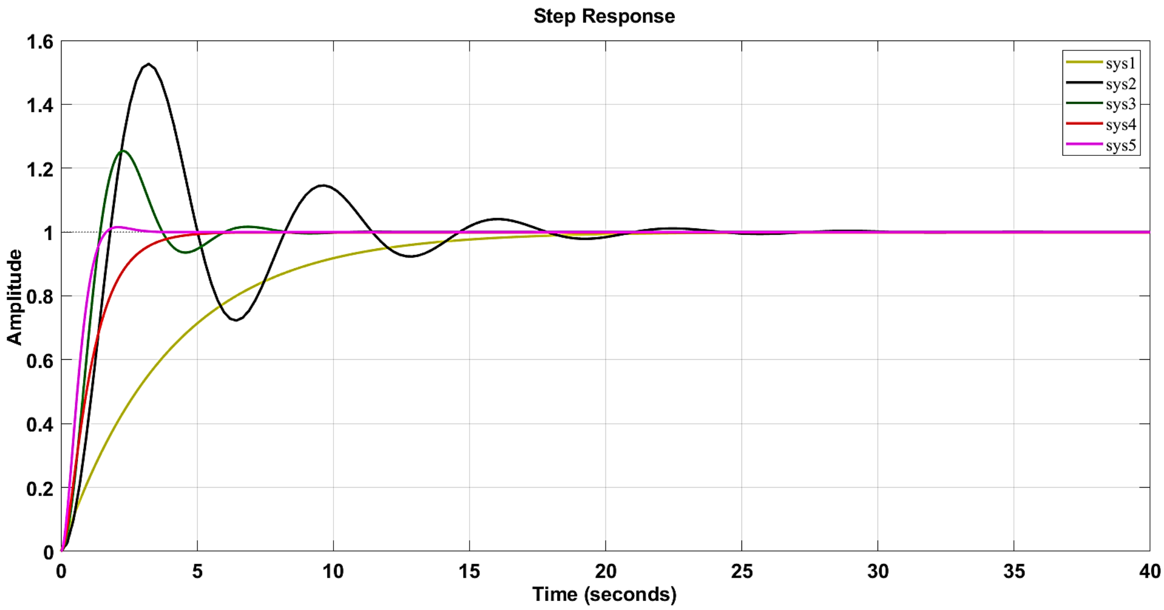


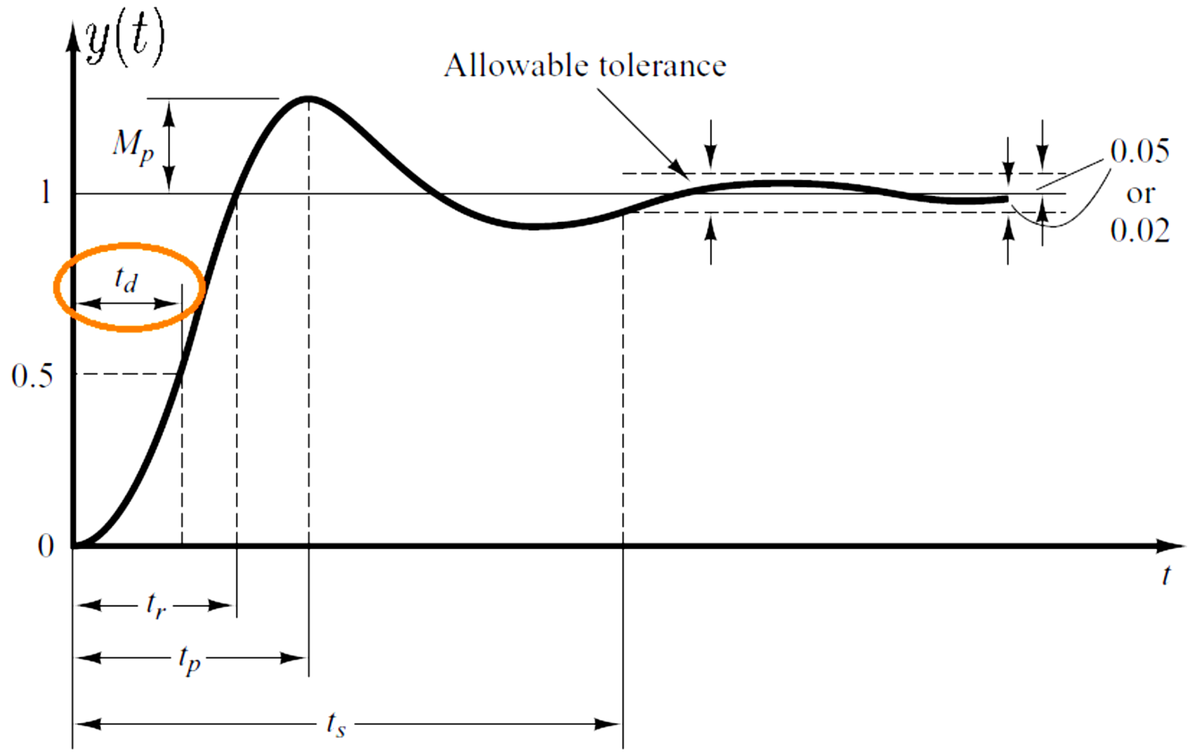
Figure 5. Step Response for different values of ζ and ω_n

$$y(t) = 1 - e^{-\zeta\omega_n t}\left[\cos\omega_d t + \frac{\zeta}{\sqrt{1-\zeta^2}} sin\omega_d t\right]$$

According to the previous equation and the step response shown in figure 5, its obvious that:

- If ζ increases the damping is increases to the response.
- If ζ decreases the damping is decreased and the system begins to oscillate.
- If ω_n increases the oscillation frequency will increase.

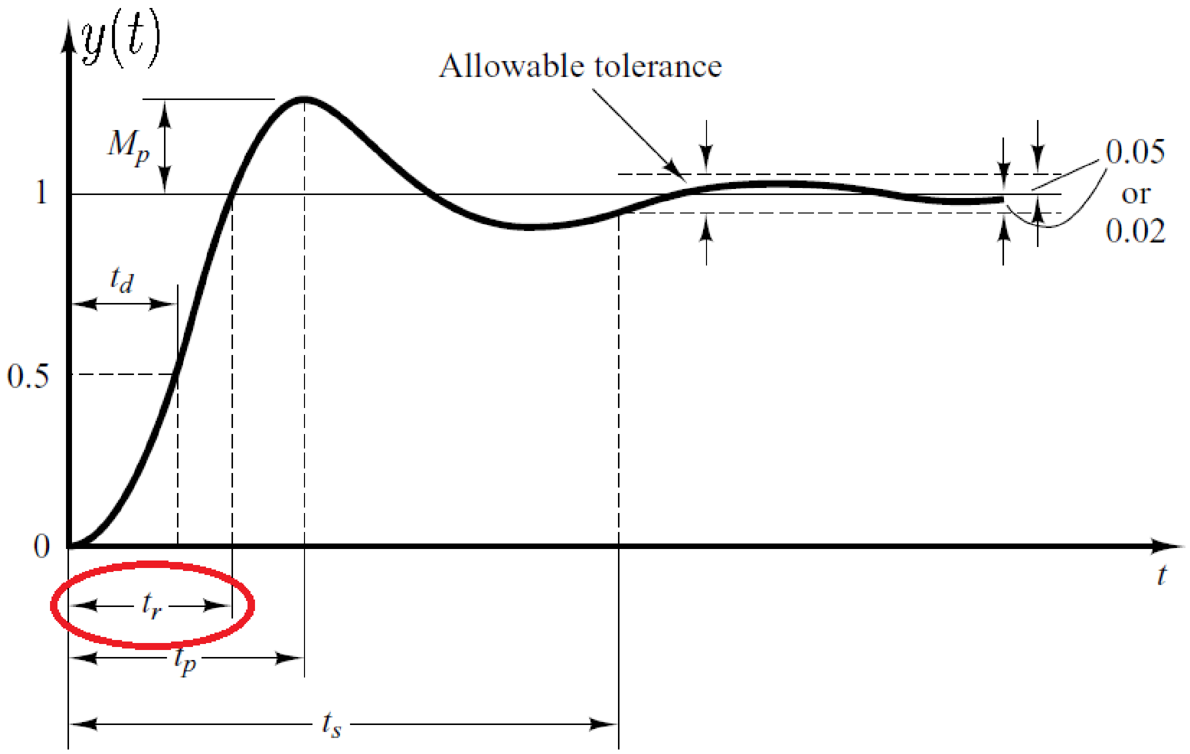
Second Order System Time Domain Specifications



Time Delay, td: It is the time required for the response y(t) to reach half of the final value.

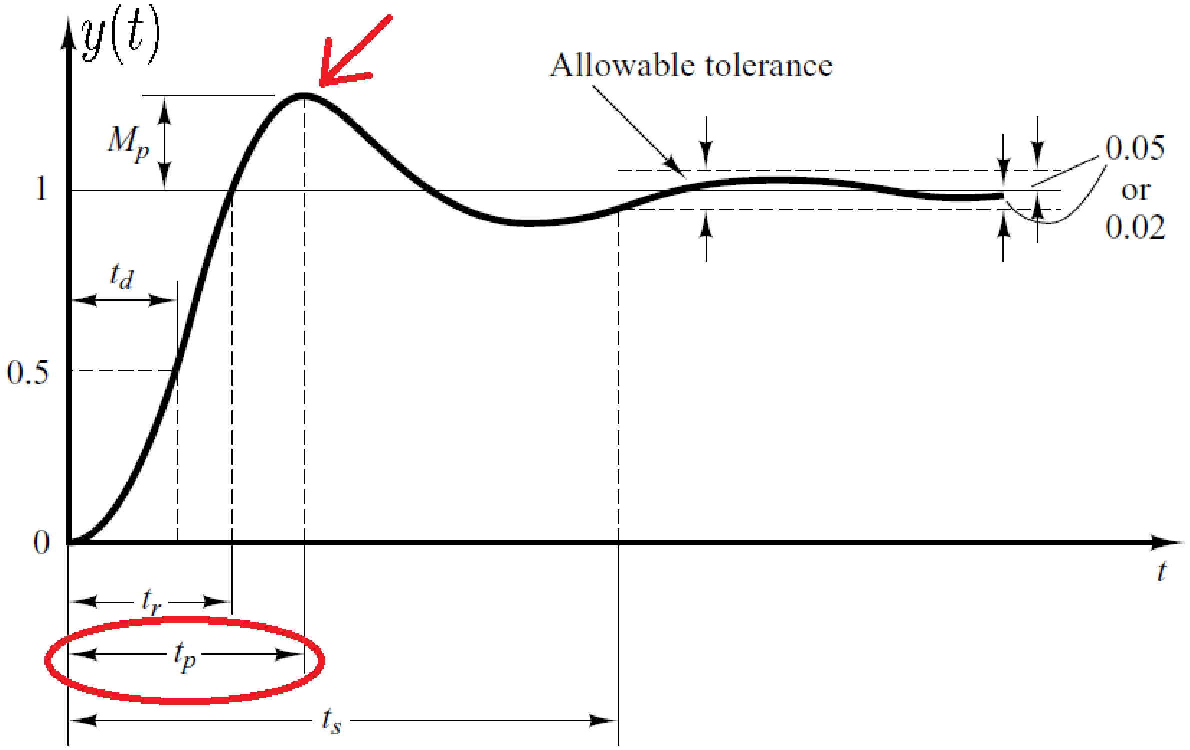
Rise Time, tr: It is the time required for the response to rise from:

0% to 100% of its final value for the under-damped system.
10% to 90% of its final value for the over-damped system.



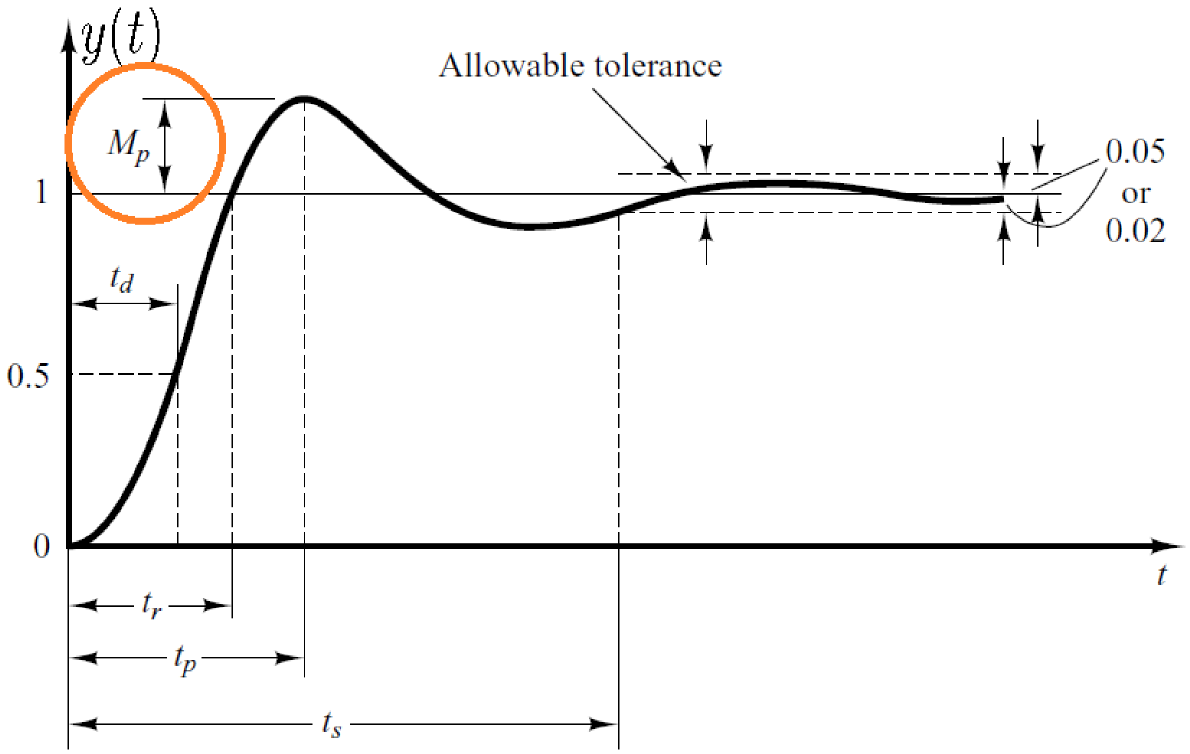
$$t_r = \frac{\pi - \theta}{\omega_d} = \frac{\pi - \theta}{\omega_n\sqrt{1 - \zeta^2}}$$

Peak Time, tp: It is the time required for the response to reach the first peak of the overshoot.



$$t_p = \frac{\pi}{\omega_d} = \frac{\pi}{\omega_n\sqrt{1-\zeta^2}}$$

Maximum Overshoot, Mp: It is the maximum peak value of the response curve measured from unity.

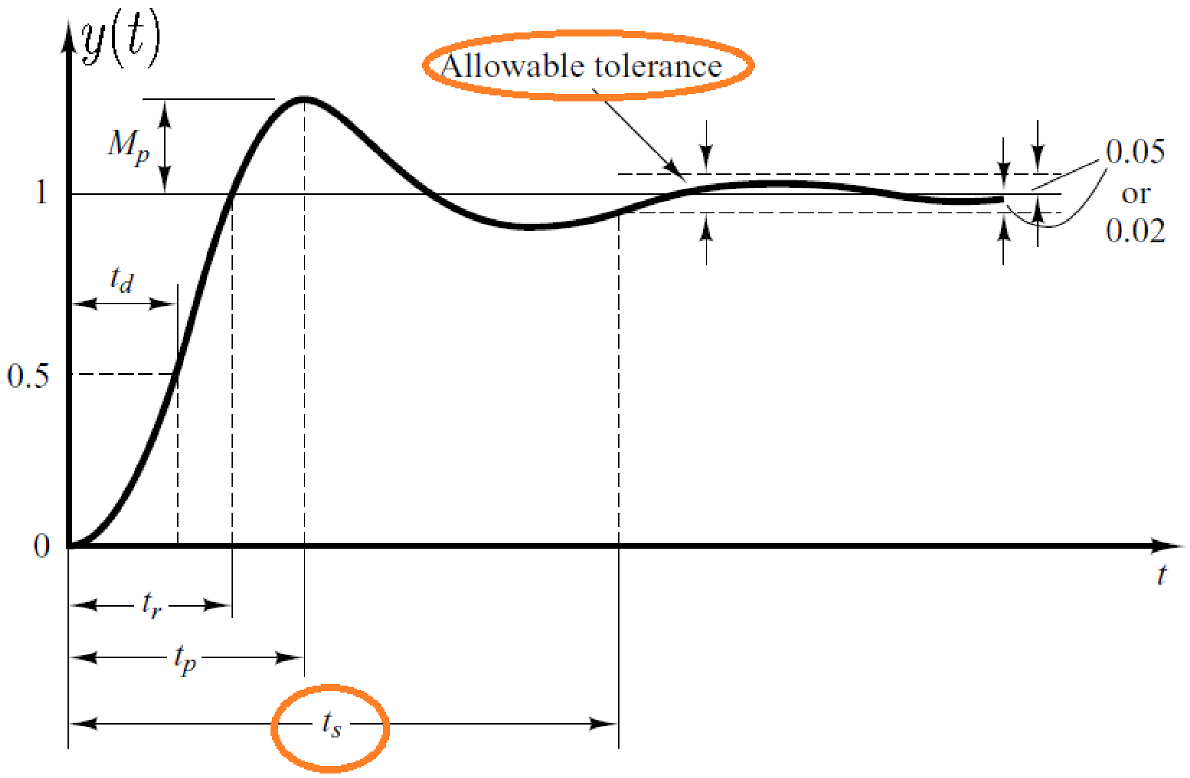


$$M_p = e^{-\frac{\pi\zeta}{\sqrt{1-\zeta^2}}} * 100$$

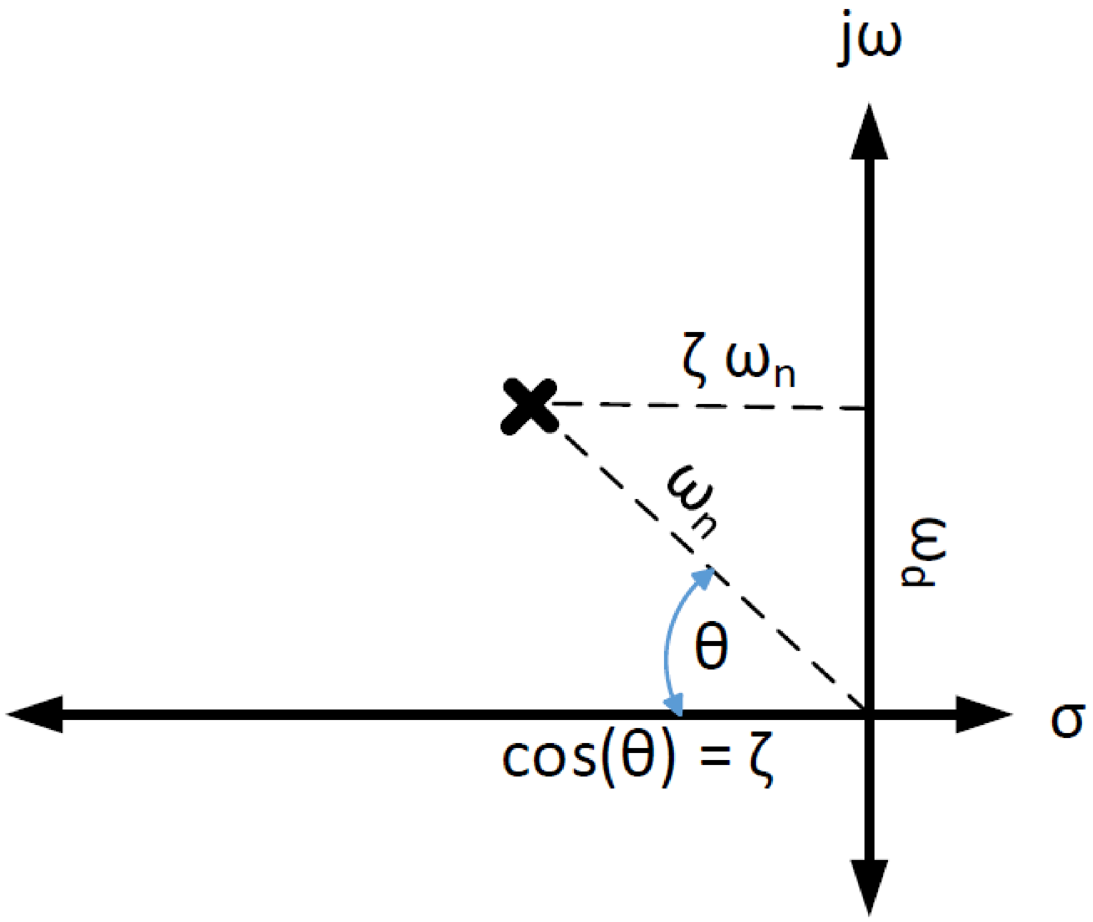
Note: If the steady-state value is not 1, the maximum percent overshoot is used:

Maximum Percent Overshoot$= \frac{y(t_p)-y(\infty)}{y(\infty)} * 100$

Settling Time, ts : It is the time required for the response curve to reach and stay within a range about the final value of size specified by absolute percentage of the final value (usually 2% or 5%).



$t_s = 4/\zeta\omega_n$ (2% criterion)
$t_s = 3/\zeta\omega_n$ (5 % criterion)



$$t_r = \frac{\pi - \theta}{\omega_d} = \frac{\pi - \theta}{\omega_n\sqrt{1-\zeta^2}}$$

$$t_p = \frac{\pi}{\omega_d} = \frac{\pi}{\omega_n\sqrt{1-\zeta^2}}$$

$$M_p = e^{-\frac{\pi\zeta}{\sqrt{1-\zeta^2}}} * 100$$

$t_s = 4/\zeta\omega_n$ (2% criterion)

$t_s = 3/\zeta\omega_n$ (5 % criterion)