Bilad Alrafidain University College

Electric Power Techniques Engineering Department

Control Systems Analysis

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Control Systems Analysis

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Lecture Five

Time Domain Analysis

Introduction to Time Domain Analysis

Absolute stability is a basic requirement of all control systems. In addition, good relative stability and steady state accuracy are also required of any control system whether continuous time or discrete time. Consider the following continuous time control system:

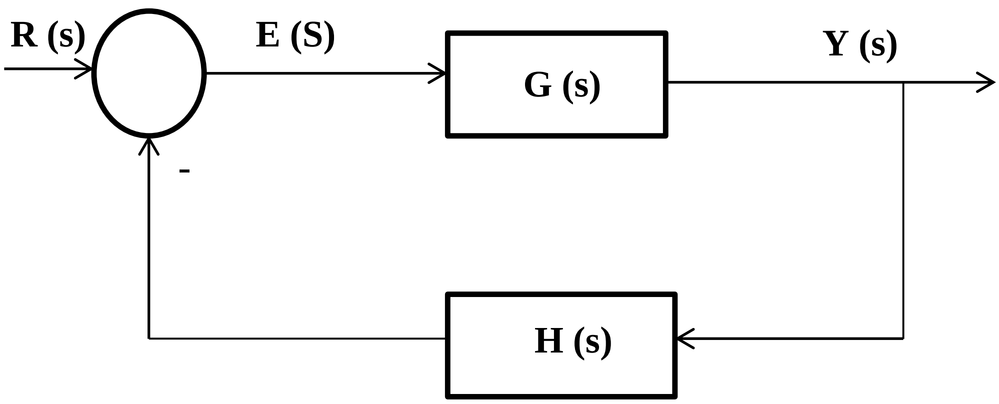


Figure 1 : Continuous time control system

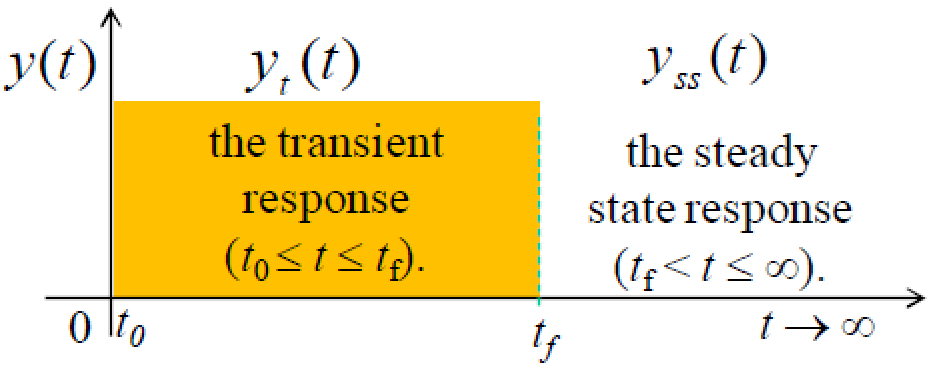
The time response of the system output $y(t)$ may be written as:

$$y(t) = y_t(t) + y_{ss}(t)$$

The transient response refers to that portion of the response due to the closed loop poles of the closed loop system.

The steady state response refers to that portion of the response due to the poles of the input or forcing function.

Transient response specifications of continuous time system



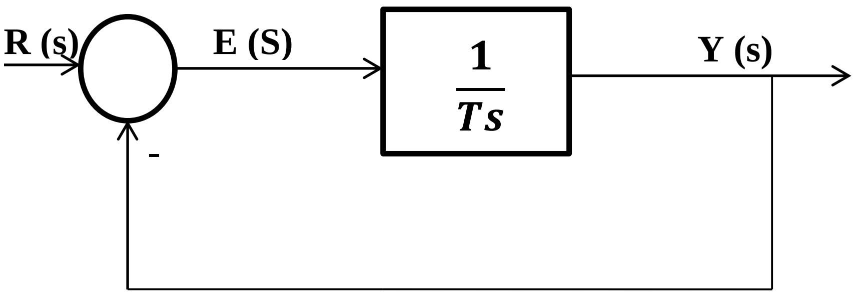
Due to the classes of dominant pole there are two modes (behaviours) occur in the transient response of continuous time system:

- **a.** First order behaviour; when the **dominant pole** is simple (has no imaginary part).
- **b.** Second order behaviour; when the **dominant pole** is complex one.

Since the standard behaviour (desired behaviour for tracking control problems) is the second order behaviour, it is enough to study it and give special attention to its specifications.

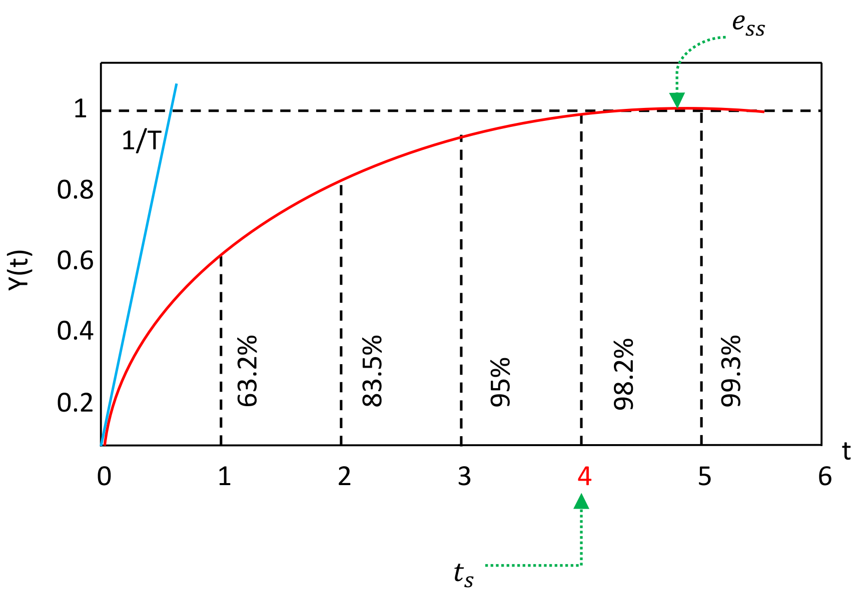
The **poles** near to the $j\omega$ axis are called the **dominant poles**. Or, get the closed-loop TF from Open loop TF. Determine the **poles** of the denominators. The **poles** which have very small real parts or near to the $j\omega$ axis have small damping ratio. These **poles** are the **dominant poles** of the system.

First order behaviour specifications: Consider the first order system given in block diagram.



$$\frac{Y(s)}{R(s)} = T(S) = \frac{1}{Ts+1}$$

$$P(s) = s + \frac{1}{T} = 0$$



The time constant T.

The slope of the tangent line at (t = 0) is 1/T.

Settling time $(t_s) = 4$ for 2% error.

Steady state errors $(e_{ss}) = \lim_{s \to 0} \frac{Ts}{(Ts+1)} = 0$

A general second-order system is characterized by the following transfer function:

$$G\ (s) = \frac{1}{\frac{1}{\omega_n^2}s^2+2\frac{\zeta}{\omega_n}s+1} = \frac{\omega_n^2}{s^2+2\omega_n\zeta s+\omega_n^2} : \text{Standard form}$$

Where:

ω_n (Undamped Natural Frequency): The frequency of the system oscillations without damping.

ζ (Damping ratio): It measures the degree of resistance to change in the system output.

For Example

$$G\ (s) = \frac{4}{s^2+2s+4}$$

$$\omega_n^2 = 4 \Rightarrow \omega_n = 2\ rad/sec$$

$$2\omega_n\zeta = 2 \Rightarrow \zeta = 0.5$$

Consider a second order system in standard form with two poles, p1 and p2:

$$\frac{\omega_n^2}{s^2 + 2\omega_n \zeta s + \omega_n^2}$$

$p1 = -\omega_n \zeta + \omega_n \sqrt{\zeta^2 - 1}$ And $p2 = -\omega_n \zeta - \omega_n \sqrt{\zeta^2 - 1}$

According to the value of ζ a second-order system can be set into one of the four categories:

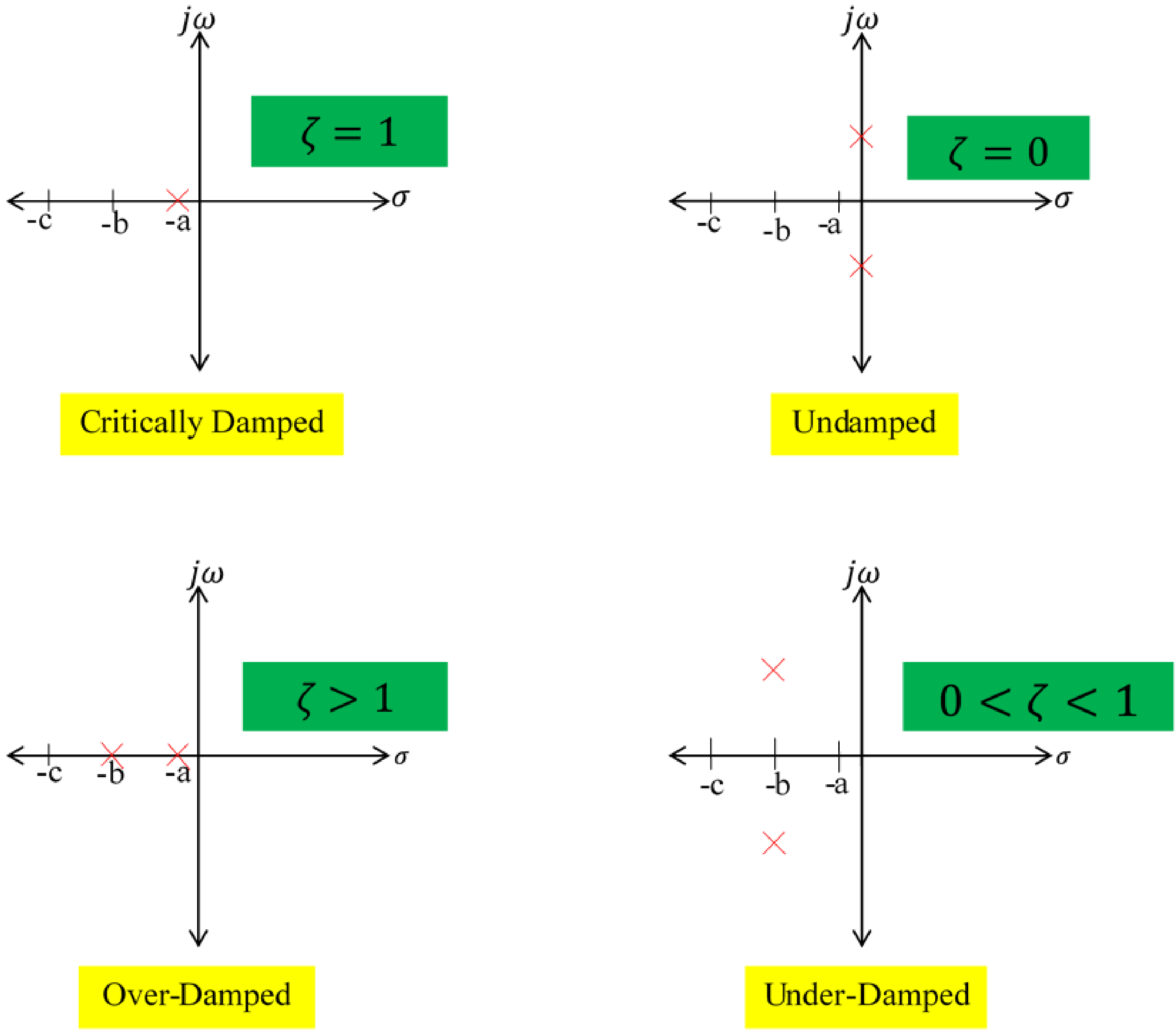


Figure 2 : Second-order system four categories