# Switching surface design via linear matrix inequality and Ackermann’s formula for a linearized inverted pendulum system

Cite as: AIP Conference Proceedings **2386**, 040017 (2022); https://doi.org/10.1063/5.0066807
Published Online: 11 January 2022

Ibrahim Ismail Ibrahim Al-Nuaimi

# Switching Surface Design via Linear Matrix Inequality and Ackermann's formula for a Linearized Inverted Pendulum System

Ibrahim Ismail Ibrahim AL-NUAIMI

*Electric Power Techniques Engineering Department, Bilad Alrafidain University College, Diyala, Iraq*
Corresponding author: Ibrahim27@bauc14.edu.iq

**Abstract :** One of the most important steps to design sliding mode control (SMC) is to design switching surface. Many methods have been developed to design the SMC. Two different approaches for the design of switching surface for a linearized inverted pendulum system have been presented in this paper. In the first method (Switching surface design via Linear Matrix Inequality (LMI)) a unique design method is proposed, which characterizes linear switching surface via LMI gave the designer some usefulness in the computational aspect, so that the switching surface for even higher-order systems can be easily calculated through this method. In the second method (Switching surface design via Ackermann's formula) which proposes a scalar sliding mode control design depends on the desired eigenvalues and the controllability matrix to achieve the desired sliding mode control performance with respect to its flexibility of solution.

**Keywords:** Sliding Mode Control, Switching surface, Linear Matrix Inequality, Ackermann's formula.

## INTRODUCTION

The Variable Structure System (VSS) is a functional system whose constructing alters in correspondence with the existing value of its condition. A (VSS) can be noticed as a system composed of separate structures jointly with an exchange logic between each of the structures. A VSS consists of a group of continuous subsystems with a suitable transforming logic thus the control activities are interrupted functions of the system condition, disturbances and reference input. The sliding mode became the basic operation mode in so-called (VSS). The main subject of this paper is to give a key point to studying and understanding the variable structure control systems (VSCs) with sliding mode control (SMC). The designing of SMC system is considered based on different methods to see the benefits and flexibility of each method as well as to achieve one similar performance for the same system in the presence of measurable and matched disturbance. The entire concept of designing the switching surface was presented with mathematical aspects including the simulation results for each approach. The SMC expression appeared in the context of VSS, specially speaking relay systems, then became the main procedure method for such a systems. In practice, all design modes for VSS are based on the studied introduction of sliding modes. The conception of SMC appeared in the Russian scientific studies in the late of 1950s. Nonetheless, the vibration control of a DC generator of an airplane by V. Kulebakin (1932) and the utilization of transfers for controlling the course of a boat by Nikolski (1934) can likewise be considered as contemporary sliding mode control [17]. It was Emelyanov who initially spotted that because of adjusting the construction over the span of controlling a cycle, the properties could be accomplished which were not characteristic in any of the individual designs [20]. The research paper by Utkin (1977) presented this idea in the English writing [16]. Ryan and Corless [11] proposed the extreme limitations and asymptotic steadiness of a group of uncertain plants (1984). Burton and Zinober [1] used the Continuous approximation of variable structure control for the smoothness of the control scheme (1986). Spurgeon and Davies [14] proposed the robust sliding mode for the plants which operating under the unmatched uncertainty (1993). The full state information is required for SMC, which is always not easy to obtain so that an observer is needed for the prediction of the system states [10.13.22]. Furat and

*3rd International Scientific Conference of Alkafeel University (ISCKU 2021)*
AIP Conf. Proc. 2386, 040017-1–040017-8; https://doi.org/10.1063/5.0066807
Published by AIP Publishing. 978-0-7354-4150-7/$30.00

Eker (2014) proposed Second-order integral SMC and the stability and robustness properties of the proposed controller are proved by means of Lyapunov stability theorem [4]. In (2016) Furat and Eker proposed the chattering-eliminated adaptive SMC and the proposed controller is compared experimentally using an electromechanical system with five different conventional sliding-mode controllers [5]. Jedda and Douik (2018) proposed a discrete-time SMC that is sophisticated for the inverted pendulum system under-actuated system linearized around its unstable equilibrium position [7]. Shokouhi and Markazi (2020) proposed three different approaches for switching functions by a new Continuous approach of Sign function instead of the conventional one, to cope the unwanted chattering phenomena in the system response [12]. Compared with the other types of control action SMC have the advantage of fixed insensitivity to different types of variations and disturbance. Therefore, the SMC is exceedingly sophisticated in the works related to the control of pendulum [2]. This Paper is organized as follows. In section 2, criteria of SMC is given. Section 3, shows the two different approach of designing switching surface for linearized inverted pendulum system. Section 4, shows the designing Results and Discussion. The conclusion is given in section 5.

## CRITERIA OF SLIDING MODE CONTROL

The SMC strategy is one of the most effective approaches to build robust controllers for complex nonlinear dynamic systems which affected by the uncertainties [21]. The SMC system is a group of continuous subsystems together by utilizing a very fast switching control action, which forced the state to be oriented towards a certain surface called switching surface.

Consider the following second order nonlinear system, n=2

$$\dot{x}_1 = f_1(x_1, x_2, u) \quad (a)$$
$$\dot{x}_2 = f_2(x_1, x_2, u) \quad (b)$$

Where, $x = (x_1, x_2) \in$Rn is state variable. If a scalar control u is considered, then the switching surface S(x) is scalar and its discontinuity points $S(x) = \{x \in R^n \mid S(x) = 0\}$ are a line in the state space. To design the controller via [18].

$$u = \begin{cases} u^+ (x,t) & s(x) > 0 \\ u^- (x,t) & s(x) < 0 \end{cases} \quad (c)$$

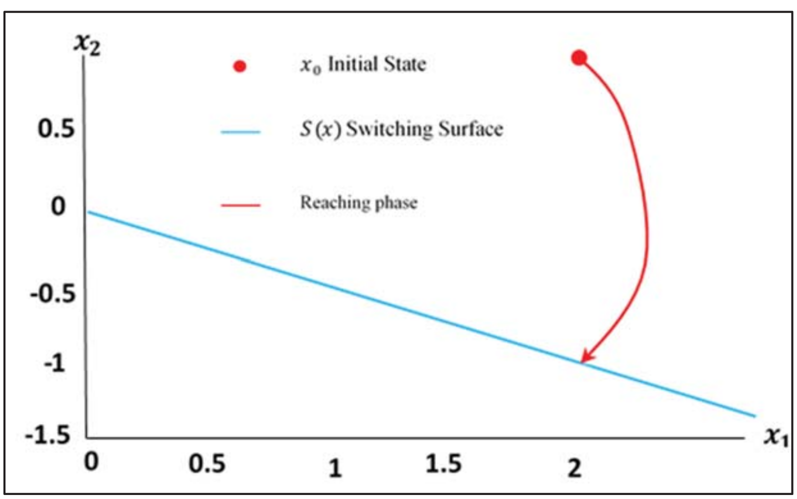



**FIGURE 1.** Reaching phase and Sliding Phase.

The controller u (x) is selected in a way that the tangent vectors of the condition path are directed across the switching surface S(x). After reaching S(x), the condition is obliged back onto the S(x) whenever a deviation occurs. Supposing unlimited fast transforming the condition will move along the S(x) after a finite period, such a movement is known as sliding mode. This is the ideal motion of the system. In all actual implementations, the system path deviates from the S(x) because of the chattering phenomena. The conception, explained for the scalar control case, can be extended in full swing to the vector case. The concept of (SMC) is to force the system condition to remain on the switching surface where the system will show eligible lineaments. The system conditions are obliged to reach the Switching surface S(x)=0 from any initial condition according to the (SMC) concept. The convergence of S(x) to zero is done in a finite time [3]. After reaching S=0, it guarantee that the control impact is able of keeping the system condition on the switching surface. The designing of sliding mode controller is divided into two main parts: First part is designing the sliding surface to obtain the wanted dynamic conduct (stability to the equilibrium point). Second part is the designing of discontinuous control law, which provides the attractivity to the system state when it is near the sliding surface and provides the stability to the closed-loop system when it is on the sliding surface. The main advantages, which the designer obtain from the system when the system is operating in the sliding mode, are the system is robust to the effect of matched uncertainties and the system performance is dominated by reduced set differential equations [23].

## METHODS

### Linear Matrix Inequality (LMI)

This section proposes an unrivaled design method of the switching surface, which portray a linear switching surface in terms of the LMI's. This approach ensures that the sliding mode functions features are fully steady and independent of different types of uncertainties [6]. The main advantages of this approach are offering additional design flexibility and some ease in the computational aspect so that the switching surface for extensive systems can be readily achieved [8.15].

**EXAMPLE 1.** Consider the linearized inverted pendulum system described by Equation (4); sliding mode control will be designed according to the following.

The initial states are: $x_1(0) = -15^o$, $x_2(0) = 0$, $x_3(0) = 5$, $x_4(0) = 0$

The desired states are: $x_1(0) = 0$, $x_2(0) = 0$, $x_3(0) = 0$, $x_4(0) = 0$

$$\begin{bmatrix} \dot{X}_1 \\ \dot{X}_2 \\ \dot{X}_3 \\ \dot{X}_4 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & -0.1818 & 2.6727 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & -0.4545 & 31.1818 & 0 \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \\ X_3 \\ X_4 \end{bmatrix} + \begin{bmatrix} 0 \\ 1.8182 \\ 0 \\ 4.5455 \end{bmatrix} (u + \phi) \quad \text{(i)}$$

Where $\phi = 0.3sin(t)$ is a measurable and matched disturbance.

Designing the switching surface *S(x)* based on LMI as follows:

$$S(x) = Cx = B^T P_x \quad \text{(ii)}$$

Where p $\in R^{nxn}$ is positive matrix and B is the input matrix. To solve for the matrix p, the system controller designed as:

$$u_t(x) = u_1(x) + u_s(x) \quad \text{(iii)}$$

Where $u_t(x)$ is the total controller, $u_1(x)$ is the linear feedback controller *(K* $\in R^{1x4}$ ) vector matrix and $u_s(x)$ is the sliding controller.

$$u_1(x) = -K\,x \quad \text{(iv)}$$

Consider the following Lyapunov function:

$$V(x) = x^T P x \quad \text{(v)}$$

Rewriting it as:

$$V(x) = [X_1 \quad X_2 \quad X_3 \quad X_4]\, P \begin{bmatrix} X_1 \\ X_2 \\ X_3 \\ X_4 \end{bmatrix},\ V(x) = P\,(x^2{}_1 + x^2{}_2 + x^2{}_3 + x^2{}_4) \quad \text{(vi)}$$

The derivative of $V(x)$ is

$$\dot{V}(x) = P(2x_1\dot{x}_1 + 2x_2\dot{x}_2 + 2x_3\dot{x}_3 + 2x_4\dot{x}_4) \quad \text{(vii)}$$

Finally the time derivative of Lyapunov function is

$$\dot{V}\,(x) = 2\,x^T\,P\,\dot{x} \quad \text{(viii)}$$

Where the closed loop system is

$$\dot{x} = A\,x + \ B\,u \quad \text{(ix)}$$

Substituting Equation (6) in Equation (12), we have

$$\dot{x} = A\,x + B\big(-K\,x + \,u_s(x)\big) = (\,A\, - BK)x + B\,u_s(x) \quad \text{(x)}$$

$$\dot{x} = \overline{A}\,x + \,B\,u_s(x) \quad \text{(xi)}$$

Where $\overline{A}$ is the closed loop matrix for $u = -Kx$ and

$$\overline{A} = A - BK \quad \text{(xii)}$$

Substituting Equation (14) in Equation (10), we have

$$\dot{V}\,(x) = 2\,x^T\,P\,(\overline{A}\,x + \,B\,u_s(x)\,) = 2\,x^T\,P\,\overline{A}\,x \;\; + 2\,x^T\,PB\,u_s(x) \quad \text{(xiii)}$$

When$(t \geq \,t_0)$ , there exists $S(x) = \,B^T\,Px = 0$. Therefore

$$s^T(x) = x^T\,PB = 0 \quad \text{(xiv)}$$

It leads to

$$\dot{V}\,(x) = 2\,x^T\,P\,\overline{A}\,x = x^T\,\left(P\,\overline{A} + \,\overline{A}^T\,P\right)x \quad \text{(xv)}$$

The condition to make $\dot{V}\,(x)\; < 0$ is

$$\left(P\,\overline{A} + \,\overline{A}^T\,P\right)\; < 0 \quad \text{(xvi)}$$

Multiplying above inequality Equation (19) by $P^{-2}$

$$\overline{A}\,P^{-1} + \,P^{-1}\overline{A}^T\;\; < 0 \quad \text{(xvii)}$$

Let $P^{-1} = M$ , then

$$\overline{A}\,M + \,M\overline{A}^T\;\; < 0 \quad \text{(xviii)}$$

Substituting Equation (15) in the above inequality Equation (21), we have

$$(A - BK)\,M + \,M(A - BK)^T\;\; < 0 \quad \text{(xix)}$$

$$AM - BKM + MA^T - MK^TB^T\;\; < 0 \quad \text{(xx)}$$

Let $L = KM$

$$AM - BL + MA^T - B^TL^T \quad \text{(xxi)}$$

The matrix $M$ can be obtained by solving the inequality defined in Equation (24) via MATLAB M-File

$$M = \begin{bmatrix} 4.2526 & -1.1725 & 0.4474 & -0.1400 \\ -1.1725 & 6.0546 & 0.0387 & 4.0537 \\ 0.4474 & 0.0387 & 0.2850 & -0.7053 \\ -0.1400 & 4.0537 & -0.7053 & 7.3276 \end{bmatrix},\; P = \begin{bmatrix} 0.4768 & 0.3140 & -1.5733 & -0.3160 \\ 0.3140 & 0.5440 & -1.7023 & -0.4588 \\ -1.5733 & -1.7023 & 11.1131 & 1.9813 \\ -0.3160 & -0.4588 & 1.9813 & 0.5750 \end{bmatrix}$$

$$L = [4.0820 \quad 0.9239 \quad 3.4366 \quad -4.6933],\; K = [-1.6871 \quad -1.9123 \quad 20.8971 \quad 2.3966]$$

$$C = B^TP = [-0.8656 \quad -1.0964 \quad 5.9110 \quad 1.7793]$$

Now the switching surface will be:

$$S(x) = Cx = -0.8656x_1 - 1.0964x_2 + 5.9110x_3 + 1.7793x_4$$

The control law $u(x)$ can be obtained through the following steps:

The general formula for exponential reaching law is

$$\dot{S}\,(x) = -\eta\;sign\big(\,S(x)\big) - k\,S(x) \quad \text{(xxii)}$$

Where $\eta > 0, k > \;0$ and $\dot{S}\,(x) = -k\,S(x)$is the exponential term, can be solved as:

$$\frac{dS(x)}{dt} = -k\,s(x) \Rightarrow \frac{dS(x)}{S(x)} = -kdt$$

$$\int \frac{dS(x)}{S(x)} = -k\int dt \quad \Longrightarrow \quad ln\,S(x) = -kt$$

Leads to $S(x) = S(0)\,e^{-kt}$

By adding the proportional rate term $-k\,S(x)$ to this reaching law forces the state to approach the switching surface faster when $S(x)$ is enough large.

It can be shown that the reaching time for $X$ state is move from an initial state $x(0)$ to the switching surface $S(x) = Cx = 0$ is finite and given by $T = \frac{1}{k}\;\; ln\frac{k\,|s| + \eta}{\eta}$

To obtain the sliding mode controller we have the direct switching function approach given in Equation (3).

Where

$$\dot{S}\,(x) = C\dot{x} \quad \text{(xxiii)}$$

From equating the two equations, Equation (25) and Equation (26), we have

$$-\eta\;sign\big(\,S(x)\big) - k\,S(x) = C\dot{x} \quad \text{(xxiv)}$$

Where $\dot{x} = Ax + Bu$

$$-\eta\;sign\big(\,S(x)\big) - k\,S(x)\;\; = C\,(Ax + Bu)$$

$$-\eta\, sign\big(S(x)\big) - k\, S(x) = CAx + CBu$$

Then the nonlinear sliding mode controller can be gained as

$$u_s(x) = \big(-\eta\, sign\big(S(x)\big) - k\, S(x) - CAx\big)\,(CB)^{-1} \quad \text{(xxv)}$$

In addition, the linear sliding mode controller can be gained as follows

$$u_1(x) = -Kx \quad \text{(xxvi)}$$

Then the total sliding mode controller will be equal to:

$$u_t(x) = u_1(x) + u_s(x) \quad \text{(xxvii)}$$

$$CB = 6.0942,\ CB^{-1} = 0.1641$$

$$CA = [0 \quad -1.4750 \quad 52.5506 \quad 5.9110]$$

The positive gains of the nonlinear sliding mode controller are selected as follows:

$\eta = 5$ , $k = 2$

$u_1(x) = -Kx = 1.6871x_1 + 1.9123x_2 - 20.8971x_3 - 2.3966x_4$

$u_s(x) = \big(-\eta\, sign\big(S(x)\big) - k\, s(x) - CAx\big)\,(CB)^{-1}$

$u_s(x) = (-5\, sign\, S(x) - 2S(x) + 1.4750x_2 - 52.5506\, x_3 - 5.9110x_4)\, 0.1641$

$u_s(x) = (-0.8205\, sign\, S(x) - 0.3282S(x) + 0.2420x_2 - 8.6235\, x_3 - 0.9699x_4)$

$u_t(x) = (1.6871x_1 + 1.9123x_2 - 20.8971x_3 - 2.3966x_4) + (-0.8205\, sign\, S(x) - 0.3282S(x) + 0.2420x_2 - 8.6235\, x_3 - 0.9699x_4)$

$u_t(x) = -0.8205\, sign\, S(x) - 0.3282S(x) + 1.6871x_1 + 2.1543x_2 - 29.5206x_3 - 3.3665x_4$

## Ackermann's Formula

In the control system design Ackermann's formula is an approach used to solving the problem of pole allocation, and it's also a proper method to mark a linear state-feedback control law in a certain feature which results in the closed-loop system with required eigenvalues [9]. The SMC equation is a linear one and rely on the switching surface coefficients, a similar task emerge in the design of SMC for the linear system with a linear surface [19]. The design of scalar SMC based on Ackermann's formula to obtain a discontinuity plane equation in outright shape as well as in terms of the main system.

**EXAMPLE 2.** Consider the linearized inverted pendulum system described by Equation (31); sliding mode control will be designed according to the following.

The initial states are: $x_1(0) = -15^o$, $x_2(0) = 0$, $x_3(0) = 5$, $x_4(0) = 0$

The desired states are: $x_1(0) = 0$, $x_2(0) = 0$, $x_3(0) = 0$, $x_4(0) = 0$

$$\begin{bmatrix} \dot{X}_1 \\ \dot{X}_2 \\ \dot{X}_3 \\ \dot{X}_4 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & -0.98 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \\ X_3 \\ X_4 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \\ 0 \\ 1 \end{bmatrix} (u + \phi) \quad \text{(xxviii)}$$

Where $f(t) = 0.5sin(3t)$ is a measurable and matched disturbance.

Designing the switching surface $S(x)$ based on Ackermann's as follows:

The desired eigenvalues of sliding motion are: $[r_1 = -1,\ r_2 = -2,\ r_3 = -3]$

$$S(x) = Cx = 0 \quad \text{(xxix)}$$

$$C = e^T P_1(A) \quad \text{(xxx)}$$

$$e^T = [0 \quad 0 \quad 0 \quad 1][B \quad AB \quad A^2B \quad A^3B]^{-1} \quad \text{(xxxi)}$$

$$P_1(A) = (A - r_1 I)(A - r_2 I)(A - r_3 I) \quad \text{(xxxii)}$$

$$C = [0 \quad 0 \quad 0 \quad 1][B \quad AB \quad A^2B \quad A^3B]^{-1}(A - r_1 I)(A - r_2 I)(A - r_3 I) \quad \text{(xxxiii)}$$

The matrix $C$ can be obtained by solving Equation (36) via MATLAB M-File.

$C = [-3.0303 \quad -5.5556 \quad 9.0303 \quad 6.5556]$

Now the switching surface $S(x) = Cx = 0$ will be:

$$S(x) = -3.0303x_1 - 5.5556x_2 + 9.0303x_3 + 6.5556x_4$$

Then the sliding mode controller based on exponential reaching law can be gained from Equation (28) as follows:

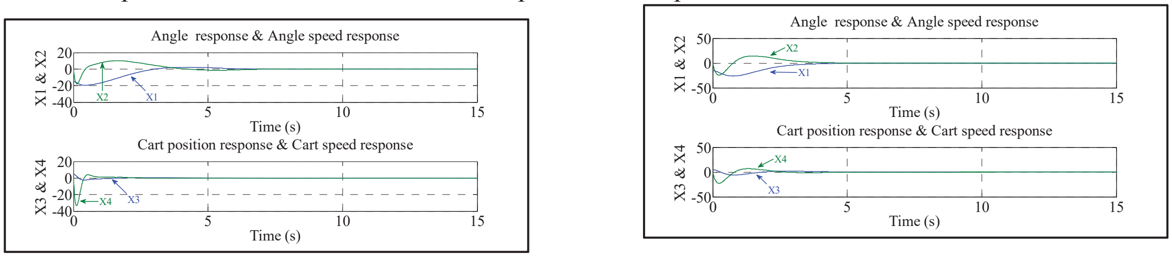
$$(CB) = 1\ ,\ (CB)^{-1} = 1$$

$$CA = [0 \quad -3.0303 \quad 12 \quad 9.0303],\ k = 2, \eta = 5$$

$$u(x) = (-5 sign S(x) - 2S(x) + 3.0303x_2 - 12x_3 - 9.0303x_4)$$

## RESULTS AND DISCUSSION

This section presents the simulation results of the previous examples.

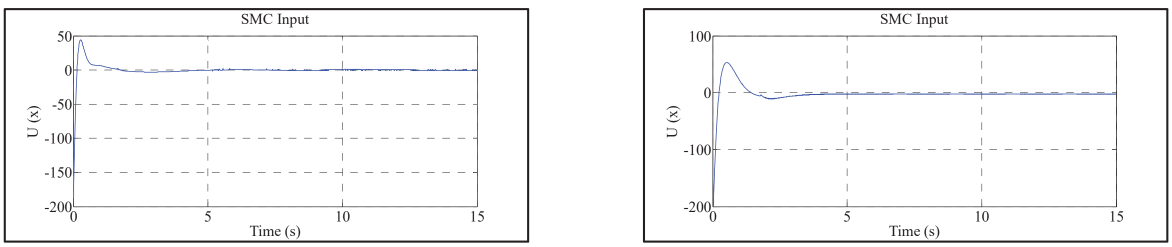



(a) LMI

(b) Ackermann's formula

**FIGURE 2.** System states

Figure 1.shows the movement of the system states from its initial conditions at (a) $[x_1(0) = -15^{\circ}, x_2(0) = 0, x_3(0) = 5^{\circ}$and $x_4(0) = 0]$ to the desired destination (switching surface) and it approaches to zero as $t \to \infty$ with settling time approximately 6 seconds for $x_1$ and $x_2$, 1.5 to 2 seconds for $x_3$ and 2.5 seconds for $x_4$ and at (b) $[x_1(0) = -15^{\circ}, x_2(0) = 0, x_3(0) = 5^{\circ}$and $x_4(0) = 0]$ to the desired destination (switching surface) and it approaches to zero as $t \to \infty$ with settling time approximately 5 seconds for $x_1$ and $x_2$, 5 seconds for $x_3$ and $x_4$.

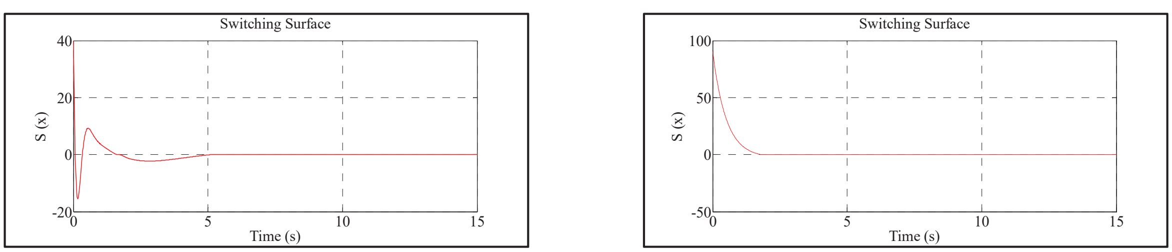



(a) LMI

(b) Ackermann's formula

**FIGURE 3.** Sliding mode controller.

Figure 3. Shows the sliding mode controller signal which has less of chattering effect at (a) and (b).




(a) LMI

(b) Ackermann's formula

**FIGURE 4.** Switching surface.

Figure 4. Presents the switching surface curve goes to zero as $t \to \infty$ with settling time about 5 seconds at (a) and settling time approximately between 1.5 to 2 seconds at (b).

## CONCLUSION

Two different approaches for the design of switching surface for linearized inverted pendulum systems have been presented in this paper. In the first method (Switching surface design based on LMI) a unique design method is proposed, which characterizes linear sliding surfaces in terms of LMIs that also gave the designer some advantages in the computational aspect, so that the sliding surfaces for even large-scale systems can be easily computed within this method. In the second method (Switching surface design based on Ackermann's formula) which proposes a scalar sliding mode control design depends on the desired eigenvalues and the controllability matrix to achieve the desired sliding mode control performance with respect to its flexibility of solution. From the previous results, we can see that the Ackermann's formula gives the result with simple way according to its procedure but its faces some difficulties compared with the LMI method, which enables the designer to easily attack various interesting problems, and for its ability of robustness against matched disturbances.

## REFERENCES


1. Burton, J.A. and Zinober, A.S., 1986. Continuous approximation of variable structure control. *International journal of systems science*, *17*(6), pp.875-885.
2. Cui, J., 2019, July. Numerical Design Method for Nonlinear Sliding Mode Control of Inverted Pendulum. In *2019 Chinese Control Conference (CCC)* (pp. 2646-2649). IEEE.
3. Derbel, N., Ghommam, J. and Zhu, Q. eds., 2017. *Applications of sliding mode control* (Vol. 79, pp. 335-337). Springer Singapore.
4. Furat, M. and Eker, I., 2014. Second-order integral sliding-mode control with experimental application. *ISA transactions*, *53*(5), pp.1661-1669.
5. Furat, M. and Eker, I., 2016. Chattering-eliminated adaptive sliding-mode control: an experimental comparison study. *Turkish Journal of Electrical Engineering & Computer Sciences*, *24*(2), pp.605-620.
6. Han Ho Choi. (1997). A New Method for Variable Structure Control System Design: A Linear Matrix Inequality Approach, *Automatica*, **33(11)**. 2089-2092.
7. Jedda, O. and Douik, A., 2018, March. Discrete-time sliding mode control for an inverted pendulum system. In *2018 International Conference on Advanced Systems and Electric Technologies (IC_ASET)* (pp. 272-276). IEEE.
8. Jinkun Liu, Xinhua Wang. (2012). Advanced Sliding Mode Control for Mechanical Systems. 1'st edition. Beijing: Tsinghua University Press, Berlin, Heidelberg: Springer-Verlag.
9. Jürgen Ackermann. (1985). Sampled-Data Control Systems. 2nd revised translation of the German 2nd edition. Berlin, Heidelberg, New York, Tokyo: Springer.
10. Luenberger, D., 1971. An introduction to observers. *IEEE Transactions on automatic control*, *16*(6), pp.596-602.
11. Ryan, E.P. and Corless, M., 1984. Ultimate boundedness and asymptotic stability of a class of uncertain dynamical systems via continuous and discontinuous feedback control. *IMA journal of mathematical control and information*, *1*(3), pp.223-242.
12. Shokouhi, F. and Davaie-Markazi, A.H., Control of Inverted Pendulum: A comparative study on sliding mode approaches.
13. Sira-Ramirez, H. and Spurgeon, S.K., 1994. On the robust design of sliding observers for linear systems. *Systems & control letters*, *23*(1), pp.9-14.
14. Spurgeon, S.K. and Davies, R., 1993. A nonlinear control strategy for robust sliding mode performance in the presence of unmatched uncertainty. *International Journal of Control*, *57*(5), pp.1107-1123.
15. Stephen Boyd, Laurent El Ghaoui, Eric Feron, Venkataramanan Balakrishnan. (1994). Linear Matrix Inequalities in System and Control Theory. Volume 15: 1st edition. Philadelphia, Pennsylvania: Society for Industrial and Applied Mathematics.
16. Utkin, V., 1977. IEEE Transactions of Automatic Control AC-12, 212} 222. *Variable structure systems with sliding mode*.
17. Utkin, V., Guldner, J. and Shijun, M., 1999. *Sliding mode control in electro-mechanical systems* (Vol. 34). CRC press.

18. Utkin. V. I. (1978). Sliding Modes and Their Application in Variable Structure Systems. 1st edition. Moscow: Mir.
19. Utkin. V. I. (1992). Sliding Modes in Control and Optimization. 1st edition. Berlin: Springer-Verlag.
20. Utkin. V. I. (2002). First Stage of VSS: People and Events. In Xinghuo Yu., Jian-Xin Xu, editors, VSS: Towards the 21st century. Volume 274: 1st edition. London: Springer. 1-32.
21. Utkin. V. I. (2009). Sliding Mode Control. In Heinz D. Unbehauen, editor, Control Systems, Robotics And Automation. Volume 13: 1st edition. Oxford, United Kingdom: EOLSS Publisher/UNESCO. 1-23.
22. Walcott, B. and Zak, S., 1987. State observation of nonlinear uncertain dynamical systems. *IEEE Transactions on automatic control*, *32*(2), pp.166-170.
23. Zak. S. H. (2003). Systems And Control. 1st edition. New York, Oxford: Oxford University Press, Inc.